ⓘ Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on February 17, 2024 1:51:26 PM,
was last modified on June 30, 2024 2:08:40 PM, and
was exported on September 7, 2026 3:31:13 PM.

Isomorphism of the de Rham cohomology with the sheaf cohomology of locally $\mathbb{R}$-constant functions

From

$$0 \to \mathbb{R} \to \cdots \to \Omega^k \xrightarrow{\mathrm{d}^k} \Omega^{k+1}$$

is an exact sequence of sheaves.

By Poincare lemma!

$$\begin{array}{ccccccccc} 0 & \to & \mathbb{R} & \to & \Omega^0 & \xrightarrow{\mathrm{d}^0} & Z\Omega^1 & \to & 0 \\ 0 & \to & Z\Omega^{k-1} & \to & \Omega^k & \xrightarrow{\mathrm{d}^k} & Z\Omega^{k+1} & \to & 0 \end{array}$$

is exact sequence of sheaves for all $k > 0$

The first exact sequences gives

```
\usepackage{tikz-cd}\usepackage{amsfonts}\usepackage{amsmath}

\begin{document}
\begin{tikzcd}

    \ & \ & {H^{p-2}(M, Z\mathrm{\Omega}^{1})} \\
    {H^{p-1}(M, \mathbb{R})} & {H^{p-1}(M, \Omega^{0})} & {H^{p-1}(M,
Z\mathrm{\Omega}^{1})} \\
    {H^{p}(M, \mathbb{R})} & \ & \

    \arrow[from=2-1, to=2-2]
    \arrow[from=2-2, to=2-3]

    \arrow[from=1-2, to=2-2, phantom, ""{coordinate, name=Z1}]
    \arrow[from=1-3, to=2-1, "\delta", rounded corners,
            to path={ -- ([xshift=10ex]\tikztostart.center)
                      |- (Z1) [pos=.25]\tikztonodes
                      -| ([xshift=-10ex]\tikztotarget.center)
                      -- (\tikztotarget)}]
    \arrow[from=2-2, to=3-2, phantom, ""{coordinate, name=Z2}]
    \arrow[from=2-3, to=3-1, "\delta", rounded corners,
            to path={ -- ([xshift=10ex]\tikztostart.center)
                      |- (Z2) [pos=.25]\tikztonodes
                      -| ([xshift=-10ex]\tikztotarget.center)
                      -- (\tikztotarget)}]




\end{tikzcd}

\end{document}
```

and the second gives for all $k > 0$

$$\begin{array}{lllll} \to & H^{p-1}(M, Z\Omega^{k-1}) & \to H^{p-1}(M, \Omega^k) & \to H^{p-1}(M, Z\Omega^{k+1}) & \to \\ & H^{p}(M, Z\Omega^{k+1}) & \to H^{p}(M, \Omega^k) & \to H^{p}(M, Z\Omega^{k+1}) & \to \end{array}$$

and we know

$$H^p(M, \Omega^k) = 0$$

$$\Downarrow$$

$$\begin{array}{lllll} \to & H^{p-1}(M, \mathbb{R}) & \to 0 & \to H^{p-1}(M, Z\Omega^{1}) & \to \\ & H^{p}(M, \mathbb{R}) & \to 0 & \to H^{p}(M, Z\Omega^{1}) & \to \end{array}$$

and

$$\begin{array}{lllll} \to & H^{p-1}(M, Z\Omega^{k-1}) & \to 0 & \to H^{p-1}(M, Z\Omega^{k+1}) & \to \\ & H^{p}(M, Z\Omega^{k+1}) & \to 0 & \to H^{p}(M, Z\Omega^{k+1}) & \to \end{array}$$

$$\Downarrow$$

$$H^p(M, \mathbb{R}) \cong H^{p-1}(M, Z\Omega^1)$$

and

$$H^p(M, Z\Omega^{k-1}) \cong H^{p-1}(M, Z\Omega^{k+1})$$

$$\Downarrow$$

$$
\begin{aligned}
H^p(M, \mathbb{R}) &\cong H^{p-1}(M, Z\Omega^1) \\
&\cong H^{p-2}(M, Z\Omega^2) \\
&\dots \\
&\cong \underbrace{H^1(M, Z\Omega^{p-1})}
\end{aligned}
$$

where by the complex

$$0 \to C^0(M, \mathrm{Cl}^{p-1}) \to C^1(M, Z\Omega^{p-1}) \to C^2$$

$$H^1(M, Z\Omega^{p-1}) = \frac{\ker \delta^1}{\mathrm{im}\ \delta^0} = H^p(\Omega^\bullet(M))$$

Hence,

$$H^p(M, \mathbb{R}) \cong H^p(\Omega^\bullet(M))$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Differential forms on smooth manifolds
          - The sheaf of differential forms on open subsets of manifolds
            - Isomorphism of the de Rham cohomology with the sheaf cohomology of locally $\mathbb{R}$-constant functions

And it has 1 siblings.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Differential forms on smooth manifolds
          - The sheaf of differential forms on open subsets of manifolds
            - Isomorphism of the de Rham cohomology with the sheaf cohomology of locally $\mathbb{R}$-constant functions