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Geodesics past conjugate points in a Riemannian manifold

Given a geodesic segment $\gamma\colon [a,c] \to M$, we say that ***γ has a conjugate point*** if there is some $b \in (a,c]$ such that $\gamma(b)$ is conjugate to $\gamma(a)$ along γ, and ***γ has an interior conjugate point*** if there is such a $b \in (a,c)$.

Theorem 10.26. *Let (M,g) be a Riemannian manifold and $p,q \in M$. If γ is a unit-speed geodesic segment from p to q that has an interior conjugate point, then there exists a proper normal vector field X along γ such that $I(X,X) < 0$. Therefore, γ is not minimizing.*

injectivity of $\exp_p$

Let (M,g) be a Riemannian manifold with $p \in M$, $B_c(0) \subseteq T_pM$ be a ball where

$$\exp_p : B_c(0) \to M$$

is defined. Then $\exp_p$ is injective on $B_c(0) \iff$ it is a diffeomorphism onto its image.

[1]

> **Proposition:** Let (M,g) be a semi-Riemannian manifold, $p \in M$ and $v \in T_pM$ such that $\exp_p(v) =: q$ is defined. Then v is a critical point of $\exp_p \iff q$ is conjugate to p along the geodesic $\exp(tv)$.

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1. differential geometry - Two definitions of the injectivity radius - Mathematics Stack Exchange ↩