Info

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Divisors on compact Riemann surfaces

Let X be a compact Riemann surface.

Definition.

$$\begin{aligned}(\,) : \mathscr{M}(X)^\times &\to \mathrm{Div}(X)\\ f &\mapsto (f) := \sum_{p\in X} \mathrm{ord}_p(f)\, p\end{aligned}$$

The image is defined to be $\mathrm{pDiv}(X)$ whose elements are **principal divisors**.

> **Proposition:** The map
>
> $$(\,) : (\mathscr{M}(X)^\times, \times) \to (\mathrm{Div}(X), +)$$
>
> satisfies
>
> $$\begin{aligned}(fg) &= (f) + (g)\\ \left(\frac{1}{f}\right) &= -(f)\end{aligned}$$
>
> Thus it is a group homomorphism.

Definition.

$$\deg : \mathrm{Div}(X) \to \mathbb{Z}$$

Thus

$$0 \to \mathbb{C}^\times \hookrightarrow \mathscr{M}(X)^\times \to \mathrm{Div}(X) \xrightarrow{\deg} \mathbb{Z} \to 0$$

is a chain complex.

$$\frac{\mathscr{M}(X)^\times}{\mathbb{C}^\times} \cong \mathrm{pDiv}(X) \hookrightarrow \mathrm{Div}^0(X)$$

$$0 \hookrightarrow \mathrm{Div}^0(X) \hookrightarrow \mathrm{Div}(X) \xrightarrow{\deg} \mathbb{Z} \to 0$$

Current note has 0 direct children and 0 total descendants.

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    - Riemann surfaces
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And it has 16 siblings.

- stem
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