Info

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Dirichlet domain of group acting isometrically and finitely discontinuously

> Let
>
> $$\Gamma \leq \mathrm{Isom}(X, d)$$
>
> acts *finitely discontinuously*.

Definition. Finitely discontinuous actions on topological spaces

Let G be a group. An *action by homeomorphisms* $G \to \mathrm{Homeo}(Y)$ on a topological space Y is **finitely discontinuous** if every point $y \in Y$ has a neighborhood U_y such that $U \cap g(U)$ is nonempty for only finitely many $g \in \mathrm{im}\, G$, that is,

$$|\{g \in \mathrm{im}\, G | U_y \cap g(U_y) \neq \emptyset\}| < \infty$$

As X is Hausdorff, we have

> Let G be a group. An *action by homeomorphisms* $G \to \mathrm{Homeo}(Y)$ on a **Hausdorff** topological space Y is **finitely discontinuous**
>
> $\iff$
>
> $\mathrm{im}\, G$ has **finite stabilizers** and the action is **stably discontinuous**. Moreover, we can take the evenly covered set U_y for any $y \in Y$ to be $(\mathrm{im}\, G)_y$-stable.

Consider

Definition. Dirichlet domain

Let $\Gamma < \mathrm{Isom}(X)$ acts finitely discotinuously on X by isometries and $p \in X$ such that $\Gamma_p = \{\mathrm{Id}_X\}$. Then the **Dirichlet domain** of Γ containing p is

$$\begin{aligned} D(\Gamma, p) &:= \{x \in X | \forall \gamma \in \Gamma, d(x,p) \leq d(\gamma(x), p)\} \\ &= \{x \in X | \forall \gamma \in \Gamma, d(x,p) \leq d(x, \gamma(p))\} \\ &= \bigcap_{\gamma \in \Gamma} \{x \in X | d(x,p) \leq d(\gamma(x), p)\} \\ &= \bigcap_{\gamma \in \Gamma} \underbrace{\{x \in X | d(x,p) \leq d(x, \gamma(p))\}}_{H(\gamma, p)} \end{aligned}$$

which is a closed, convex subset of X.

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If $p \in X$ is not fixed by any $\gamma \in \Gamma \setminus \{\mathrm{Id}_X\}$, then $D_{\Gamma, p}$ is a convex, locally finite (closed) fundamental region for Γ.

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- structures based on sets
  - Metric spaces
    - Metric space isometries
      - Isometric action on a metric space
        - Dirichlet domain of group acting isometrically and finitely discontinuously

And it has 1 siblings.

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