The only compact connected Lie group that can act faithfully on a torus is itself

6.54 Proposition (Bourguignon, Yau) The only connected, compact Lie group which can act effectively on the torus T^n is a torus T^k, $k \leq n$.

PROOF Suppose that a connected compact group G acts effectively on T^n. It suffices to show that the Lie algebra $\mathfrak{g}$ of G is abelian. By averaging a metric on T^n over the action of G by integration (see the proof of 6.8) we may assume that G acts on T^n by isometries. By 5.13, $\mu(X)$ is constant on T^n if μ is a harmonic 1-form and X is a Killing field on T^n. Therefore if X and Y are Killing fields on T^n, then for each harmonic 1-form μ on T^n,

$$\mu([X, Y]) = X\mu(Y) - Y\mu(X) - d\mu(X, Y) = 0 \qquad (d\mu = 0)$$

If $\{\mu^i\}$ is a basis for the space of harmonic 1-forms on T^n, $i = 1, \ldots, n$, then $\mu^1 \wedge \cdots \wedge \mu^n$ is cohomologous to the volume form on T^n, and is therefore nonzero on some open subset of T^n. Therefore $[X, Y] = 0$ on this open set; since $[X, Y]$ is a Killing field on T^n, it is zero everywhere on T^n. But by hypothesis, G is a subgroup of the isometry group of T^n; thus (3.53) $\mathfrak{g}$ is isomorphic to a Lie subalgebra of the Killing fields on T^n, and is therefore abelian.

For an alternative proof, see [Y].

Exercise In the proof above, instead of averaging a metric on T^n over the action of G, it suffices to average a linear connection on T^n over the action of G.

Current note has 0 direct children and 0 total descendants.

- stretchy
  - The only compact connected Lie group that can act faithfully on a torus is itself

And it has 62 siblings.

- stretchy
  - Trefoil 3_1 knot
  - The only compact connected Lie group that can act faithfully on a torus is itself
  - Hecke algebras
  - $\overline{B^n}$
  - $\mathrm{BS}(1,2)$
  - $(\mathbb{C},+,\times)$
  - $D^* := D \setminus \{0\}$
  - Cantor set
  - Configuration space of N rigid points in $\mathbb{R}^3$
  - Free groups
  - $GL(2)(\mathbb{Z}) \curvearrowright T^2$
  - H^3_2
  - $[a,b]$
  - Figure 8 knot
  - Lamplighter group on $\mathbb{Z}_2$
  - $\beta\mathbb{N}$
  - $\#_2\mathbb{R}P^2$
  - $\prod_{i\in\mathbb{N}}\{1,\ldots,n_i\}$
  - $\mathbb{R}P^n$
  - $(\mathbb{Q},+,\times,<)$
  - $\mathbb{Q}_{\hat{p}}$
  - $\overline{\mathbb{Q}}$
  - $\mathbb{Q}_{\text{constr}}$
  - $\mathbb{Q}(\xi_n)$
  - $\mathcal{O}_{\overline{\mathbb{Q}}}$
  - $(\mathbb{R},+,\times,<,\sup)$
  - Homeomorphism of $\mathbb{R}^2$ that sends $\mathbb{N}$ to a countable closed discrete subset
  - $\mathbb{R}^2 \times S^1$
  - $\mathbb{R}^n$
  - $\mathbb{R} \times S^1$
  - $\mathbb{R} \times S^2$
  - S^1
  - $S^1 \wedge S^1$
  - $S^1 \times S^n$
  - S^2 and $\mathbb{C}P^1$
  - Mapping torus of the antipodal map of S^2
  - S^3
  - S^n
  - I-bundle
  - Symmetric group
  - $\mathbb{T}$

- T^2
- T_g^2
- $T_{0,0,3}^2$
- $\mathbb{R}^2 \setminus \{0\}$
- Three $T_{1,0,1}^2$ glued at their boundaries
- $T_{1,1,0}^2$
- T_2^2
- $T_g^2 \times [0,1]$
- $\#_{\mathbb{N}} T^2$
- Thompson's group
- T^n
- 3-valent tree
- 4-valent tree
- $[0,1]^{\mathbb{N}}$ with product topology
- Whitehead manifolds
- $\mathbb{Z}$
- $\mathbb{Z}[\sqrt{m}]$
- $\mathbb{Z}[X]$
- $\mathbb{Z}^{\mathbb{N}}$
- $\mathbb{Z}[i]$
- $\mathbb{Z}[i\sqrt{5}]$