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smooth $\mathbb{R}$ -manifold $(M, \mathcal{T}_M, \mathcal{A}_\infty^{\max})$

Definition. A smooth $\mathbb{R}$ -manifold is a $\mathcal{C}^\infty$ -[manifold](#) $(M, \mathcal{T}_M, \mathcal{A}_\infty^{\max})$

A $\mathcal{C}^\infty$ -**manifold** is a triple $(M, \mathcal{T}_M, \mathcal{A}_\infty^{\max})$ where

- $(M, \mathcal{T}_M)$ is a

Definition. Topological manifold

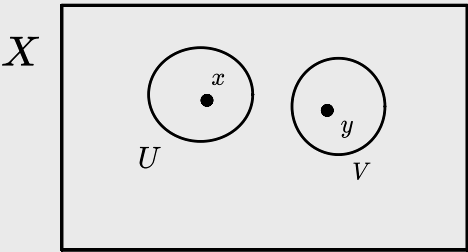
A [topological space](#) is a topological $\mathcal{C}^0$ **$\mathbb{R}$ -manifold of $\mathbb{R}$ -dimension n** for a $n \in \mathbb{Z}_{>0}$ if

- M is Hausdorff topological space

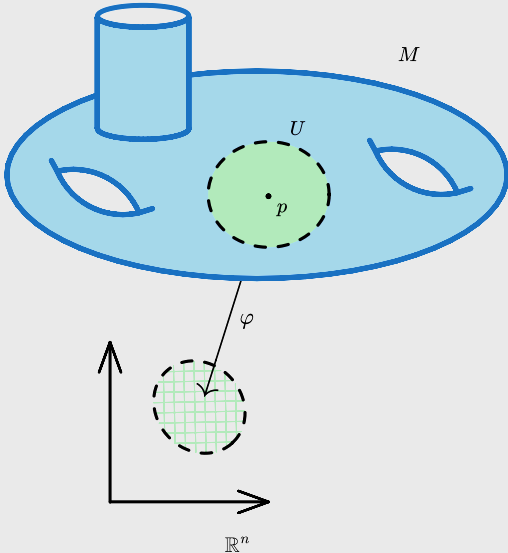
Definition. T_2 topological space or Hausdorff space

A [topological space](#) $(X, \mathfrak{T})$ is said to be T_2 **or Hausdorff** if we can "house off" every pair of points using disjoint open sets

$$\begin{aligned} \forall x, y \in X \exists U, V \in \mathfrak{T} \\ \text{st } x \in U, y \in V \\ \text{and } U \cap V = \emptyset \end{aligned}$$



- M is [second countable](#)
- (locally homeomorphic to $(\mathbb{R}^n, \mathfrak{T}_{\text{DOT}})$) for any $p \in M$, there exists



- an open subset $U \subseteq M$ with $p \in U$
- open subset $V \subseteq \mathbb{R}^n$
- and a homeomorphism $\varphi : U \rightarrow V$

- $\mathcal{A}_\infty^{\max}$ is a maximal $\mathcal{C}^\infty$ -atlas

Definition. $\mathcal{C}^\bullet$ -compatible charts

Two charts (U, x) and (V, y) are said to be $\mathcal{C}^\bullet$ -compatible if either

- $U \cap V = \emptyset$ or
- the map $y \circ x^{-1}$ is $\mathcal{C}^\bullet$
 - $\bullet = 0$
 - $\bullet = k \in \mathbb{Z}^+$
 - $\bullet = \infty$
 - $\bullet = \omega$

A $\mathcal{C}^\bullet$ -**atlas** of a [topological manifold](#) is an atlas of pairwise $\mathcal{C}^\bullet$ -compatible charts. We denote it by $\mathcal{A}_\bullet$.

A $\mathcal{C}^\bullet$ -atlas $\mathcal{A}_\bullet$ of a [topological manifold](#) is a **maximal $\mathcal{C}^\bullet$ -atlas** if for every $(U, x) \in \mathcal{A}_\bullet$ we have [all the charts](#) $(V, y) \in \mathcal{A}_\bullet$ that are $\bullet$ -compatible with (U, x) . We denote it by $\mathcal{A}_\bullet^{\max}$

Current note has 19 direct children and 82 total descendants.

- [structures based on sets](#)
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 - [Smooth Lie group action on a smooth manifold](#)
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 - [Proper Lie group actions](#)
 - [invariant Riemannian metric in a proper Lie group action](#)
 - [bord](#)

- Ring of closed manifolds upto cobordism
- TQFT on category of bordisms
- Borel measures on a smooth manifold
- G -bundles over smooth manifolds $G \curvearrowright P \rightarrow X$
- Vector bundle on smooth manifolds
 - Connection on a smooth vector bundle on a smooth manifold
 - Inducing vector bundles
 - Homotopic maps induce isomorphic vector bundles
 - Vector bundles over a contractible manifold
 - Pullback of a smooth vector bundle on a smooth manifold
 - Tangent bundle of a smooth vector bundle
- Curves in smooth manifolds
- Density on smooth manifolds
- Functions between smooth manifolds
 - Critical values of smooth maps between manifolds
 - Embedding smooth manifolds in $\mathbb{R}^n$
 - Iterations of a smooth map on a smooth manifold
 - Submersions, immersions and embeddings of smooth manifolds
 - Suspension of diffeomorphisms of smooth manifolds
- Functions from smooth manifolds to $\mathbb{R}$
 - Critical values of smooth functions to $\mathbb{R}$
 - Smooth function on a smooth manifold with two critical points
 - Equations for functions on smooth manifolds
 - General first order Cauchy problem on a smooth manifold
 - Linear first-order Cauchy problem on a smooth manifold
 - $\mathcal{C}^\infty(M)$
- Flows on a smooth manifold
 - Constants of the flow on a smooth manifold
- Locally $G \curvearrowright X$ -manifolds
- Smooth singular homology of a smooth manifold
 - $Z_{k-1}\text{sing}^\infty(M, \mathbb{Z}) \rightarrow U(1)$ differential characters
 - Dual differential forms of smooth singular chains
- Isomorphisms and automorphisms of smooth manifolds
- Lagrangian flows
- Local transformation rules
- Moduli space of Riemannian metrics on a smooth manifold
- Moduli space of quotients
- Quotient manifolds
- $\mathbb{S}$
 - k -cycles of closed manifolds
 - Duals of submanifolds
- Submanifolds of smooth manifolds
 - Immersed submanifolds of a smooth manifold
 - Isotropy of submanifolds
- Tangents on a smooth manifold
 - Cotangent space on a smooth manifold T_p^*M
 - Standard symplectic form on cotangent bundle of a manifold
 - Connection on the tangent bundle of a smooth manifold
 - Differential forms on smooth manifolds
 - Differential 1-forms on smooth manifolds
 - Closed differential 1-forms on smooth manifolds
 - When are closed differential 1-forms exact on a smooth manifold?
 - Conservative 1-forms
 - Chain complex of differential forms on a smooth manifold
 - Homotopy invariance of cohomology of differential forms
 - Exterior derivative operator on space of smooth differential forms
 - Functoriality of Forms^k on smooth manifolds
 - The integral map on space of differential forms
 - Three different relative chain complexes of differential forms
 - Differential forms relative to a smooth map
 - Chain complex of differential forms relative to a submanifold
 - The sheaf of differential forms on open subsets of manifolds
 - Isomorphism of the de Rham cohomology with the sheaf cohomology of locally $\mathbb{R}$ -constant functions
 - tensor
 - Sections of tangent tensor bundle on a smooth manifold
 - Tangent space of a smooth manifold
 - Canonical bijection of T_pM with equivalence classes of velocities
 - Tangents on smooth manifolds as derivations $\mathcal{C}^\infty(M) \rightarrow \mathbb{R}$
 - local basis of a tangent space of a smooth manifold
 - Tangent bundle TM of a smooth manifold M
 - Cotangent bundle T^*M on a smooth manifold M
 - Comparison of constructions with tangent vectors
 - Tangent frames on smooth manifolds
 - Tangent vector fields on a smooth manifold
 - Vector field on smooth manifold M as derivation of $\mathcal{C}^\infty(M)$

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- [Exponential of vector fields](#)
- [Index of a vector field](#)
- [Lie bracket of two smooth vector fields on manifold](#)

And it has 20 siblings.

- [structures based on sets](#)
 - [manifolds](#)
 - [Closed aspherical manifolds](#)
 - [ℂ-manifold](#)
 - [Hermitian manifolds](#)
 - [Kahler manifolds](#)
 - [Kahler manifolds of complex dimension 2, AKA Kahler surfaces](#)
 - [smooth ℝ-manifold](#)
 - [Principal bundles over smooth manifolds](#)
 - [Manifold with a flow](#)
 - [ℝ-manifold group \(Lie groups\)](#)
 - [Homogeneous manifold](#)
 - [Finsler manifold](#)
 - [ℝ-manifold semi-Riemannian](#)
 - [Sub-Riemannian manifolds](#)
 - [ℝ-manifold Poisson](#)
 - [ℝ-manifold symplectic](#)
 - [Multisymplectic manifolds](#)
 - [Spin ℝ-manifold](#)
 - [ℝ-manifold topological](#)
 - [Toric manifolds](#)
 - [Manifolds with compatible triples](#)