Info

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$(\mathbb{C},+,\times)$

Definition. $(\mathbb{C},+,\times)$

On the Abelian group

$$(\mathbb{C},+) := (\mathbb{R}^2,+)$$

with additive identity $0_{\mathbb{C}} := (0,0)$ we define the multiplication structure

$$\begin{gathered}\times : \mathbb{C}\times\mathbb{C} \to \mathbb{C}\\ (a,b)\times(c,d) := (ac-bd, bc+ad)\end{gathered}$$

with multiplicative identity $1_{\mathbb{C}} := (1,0)$.
By $i_{\mathbb{C}} := (0,1)$ we may write an element $(a,b) = a+ib$ where "a" means $a1_{\mathbb{C}}$.

- $1_{\mathbb{C}} := (1,0)$
- $i_{\mathbb{C}} := (0,1)_{\mathbb{R}^2}$

Then $(a,b)_{\mathbb{R}^2} = a + i_{\mathbb{C}}b$, with

- $(a+i_{\mathbb{C}}b)+(c+i_{\mathbb{C}}d) = a+c+i_{\mathbb{C}}(b+d)$
- $\times$ is associative, distributive and commutative with
  - $i_{\mathbb{C}}^2 = -1_{\mathbb{C}}$.

Take the $\mathbb{R}$-vector space $(\mathbb{R}^2,+,)$ and the $\mathbb{R}$-linear map $\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ and define

$$\begin{gathered}\times : \mathbb{R}^2\times\mathbb{R}^2 \to \mathbb{R}^2\\ \begin{bmatrix}a\\ b\end{bmatrix}\times\begin{bmatrix}c\\ d\end{bmatrix} := a\begin{bmatrix}c\\ d\end{bmatrix} + b\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}c\\ d\end{bmatrix}\end{gathered}$$

then

$$(\mathbb{R}^2,+,\times)$$

is field isomorphic to $\mathbb{C}$

$$\left(\mathbb{R}^2,+,,\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\right)$$

is a $\mathbb{R}$-vector space with a complex structure

The subalgebra

$$\left(\{a\underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{\mathrm{I}_2} + b\underbrace{\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}}_{\mathrm{Rot}_{\frac{\pi}{2}}} : a,b\in\mathbb{R}\},+,\times\right)$$

of $\mathrm{Mat}_{\mathbb{R}}(2)$ is $\mathbb{R}$-algebra isomorphic to $\mathbb{C}$.

- The complex tangent space on $\mathbb{C}$
- C^1-singular homology on $U\subseteq\mathbb{C}$
- Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
- Degree 1 rational transformations
- The complex exponential $\exp(z)$

Current note has 1 direct children and 1 total descendants.

- stretchy
  - $(\mathbb{C},+,\times)$
    - $\mathbb{C}$-linear maps $\mathbb{C}\to\mathbb{C}$

And it has 62 siblings.

- stretchy
  - Trefoil 3_1 knot
  - The only compact connected Lie group that can act faithfully on a torus is itself
  - Hecke algebras
  - $\overline{B^n}$
  - $\mathrm{BS}(1,2)$
  - $(\mathbb{C},+,\times)$
  - $D^* := D\setminus\{0\}$
  - Cantor set
  - Configuration space of N rigid points in $\mathbb{R}^3$
  - Free groups
  - $GL(2)(\mathbb{Z}) \curvearrowright T^2$

- H_2^3
- $[a,b]$
- Figure 8 knot
- Lamplighter group on $\mathbb{Z}_2$
- $\beta\mathbb{N}$
- $\#_2\mathbb{R}P^2$
- $\prod_{i\in\mathbb{N}}\{1,\ldots,n_i\}$
- $\mathbb{R}P^n$
- $(\mathbb{Q},+,\times,<)$
- $\mathbb{Q}_{\hat{p}}$
- $\overline{\mathbb{Q}}$
- $\mathbb{Q}_{\mathrm{constr}}$
- $\mathbb{Q}(\xi_n)$
- $\mathcal{O}_{\overline{\mathbb{Q}}}$
- $(\mathbb{R},+,\times,<,\sup)$
- Homeomorphism of $\mathbb{R}^2$ that sends $\mathbb{N}$ to a countable closed discrete subset
- $\mathbb{R}^2\times S^1$
- $\mathbb{R}^n$
- $\mathbb{R}\times S^1$
- $\mathbb{R}\times S^2$
- S^1
- $S^1\wedge S^1$
- $S^1\times S^n$
- S^2 and $\mathbb{C}\boldsymbol{P}^1$
- Mapping torus of the antipodal map of S^2
- S^3
- S^n
- I-bundle
- Symmetric group
- $\mathbb{T}$
- T^2
- T_g^2
- $T_{0,0,3}^2$
- $\mathbb{R}^2\setminus\{0\}$
- Three $T_{1,0,1}^2$ glued at their boundaries
- $T_{1,1,0}^2$
- T_2^2
- $T_g^2\times[0,1]$
- $\#_{\mathbb{N}}T^2$
- Thompson's group
- T^n
- 3-valent tree
- 4-valent tree
- $[0,1]^{\mathbb{N}}$ with product topology
- Whitehead manifolds
- $\mathbb{Z}$
- $\mathbb{Z}[\sqrt{m}]$
- $\mathbb{Z}[X]$
- $\mathbb{Z}^{\mathbb{N}}$
- $\mathbb{Z}[i]$
- $\mathbb{Z}[i\sqrt{5}]$