Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on February 21, 2026 6:50:54 PM,
was last modified on September 7, 2026 1:10:52 PM, and
was exported on September 7, 2026 3:09:51 PM.

$H^1_{\text{loc}}(U)$ for $U \subseteq \mathbb{R}^2$

Consider

Definition. Local Sobolev space H^1_{loc}

Let $U \subseteq \mathbb{R}^n$ be open. The **local Sobolev space** of functions with derivatives in L^2_{loc}

$$H^1_{\text{loc}}(U) := \left\{ f \in L^2_{\text{loc}}(U) \middle| \forall 1 \leq j \leq n, \overset{\int}{\partial}_j f \in L^2_{\text{loc}}(U) \right\}$$

with norms

$$\|f\|_{H^1(\mathring{K})}$$

for each compact $K \subseteq U$.

for $n = 2$.

Identifying $\mathbb{C}$ with $\mathbb{R}^2$ itself we may identify the *functions* with *endomorphisms*

$$H^1_{\text{loc}}(U) \subseteq \operatorname{HomMeas}(U, \mathbb{R}^2)$$

and

$$H^1_{\text{loc}} \cap \mathcal{C}(U) \subseteq \mathcal{C}(U, \mathbb{R}^2)$$

Definition. Let $f \in H^1_{\text{loc}} \cap \mathcal{C}(U)$ then its **Jacobian**

$$\operatorname{Jac} f := \left| \frac{\partial f}{\partial z} \right|^2 - \left| \frac{\partial f}{\partial \bar{z}} \right|^2$$

(Change of variables formula) Let $U, V \subseteq \mathbb{C}$ be open and $f : U \to V$ be a proper map in $H^1_{\text{loc}} \cap \mathcal{C}(U)$. Then

$$\forall g \in \mathcal{C}_c(V), (\deg f) \int_V g = \int_U (g \circ f) \operatorname{Jac} f$$

where

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and

$$\begin{array}{rccc} f^* : & H^2(\operatorname{sing}^\bullet_c(V, \mathbb{Z})) & \to & H^2(\operatorname{sing}^\bullet_c(U, \mathbb{Z})) \\ & \cong & & \cong \\ \deg f : & \mathbb{Z} & \to & \mathbb{Z} \end{array}$$

- Let $\operatorname{supp} g \subseteq V'(\text{open}) \subseteq V$.
  - Consider a sequence $\{f_n\} \subseteq \mathcal{C}^\infty(U)$ be such that

    $$f_n : f_n^{-1}(V') \to V'$$

    is proper of $\deg f$ for all n.
  - ... [1]

(Area and the Jacobian) Let $U, V \subseteq \mathbb{C}$ be bounded open sets and let

$$f : U \to V$$

be an orientation-preserving homeomorphism $H^1_{\text{loc}} \cap \mathcal{C}(U)$. Then for any open $W \subseteq U$ we have

$$m(f(U)) = \int_W \operatorname{Jac} f$$

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Differentiable functions
      - Distributional derivatives
        - $H^1_{\text{loc}}(U)$ for $U \subseteq \mathbb{R}^2$

And it has 4 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
    - Differentiable functions
      - Distributional derivatives
        - Fractional Sobolev spaces
        - $H^1(U)$
        - H^1_{loc}
        - $H^1_{\text{loc}}(U)$ for $U \subseteq \mathbb{R}^2$

1. John H. Hubbard - Teichmüller Theory And Applications To Geometry, Topology, And Dynamics. Volume 1 Teichmüller Theory, p.141 ↩