

Info

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$H(3,\mathbb{R})/\mathbb{Z}$

#Wiki/group/Lie

Definition.

$$H(3,\mathbb{R})/\mathbb{Z}:=\frac{\left\{\begin{bmatrix}1&a&b\\0&1&c\\0&0&1\end{bmatrix}\middle|a,b,c\in\mathbb{R}\right\}}{\left\{\begin{bmatrix}1&0&n\\0&1&0\\0&0&1\end{bmatrix}\middle|n\in\mathbb{Z}\right\}}$$

Proposition:

$$H(3,\mathbb{R})/\mathbb{Z}\cong\mathbb{R}^2\times S^1$$

with center

$$Z\left(H(3,\mathbb{R})/\mathbb{Z}\right)\cong S^1$$

The group

Definition.

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has no faithful finite dimensional representation.

Definition.

$$\begin{aligned}\mathfrak{h}(3,\mathbb{R})&\rightarrow\mathfrak{gl}(\mathcal{C}^\infty(\mathbb{R}))\\f&\overset{X}{\mapsto}(\pi i)x f(x)\\f&\overset{Y}{\mapsto}f'\\f&\overset{I}{\mapsto}-\pi i\mathrm{Id}\end{aligned}$$

$$\begin{aligned}\rho_\lambda:H(3,\mathbb{R})&\curvearrowright\mathcal{B}(\mathcal{C}^\infty(\mathbb{R}))\\f&\overset{a,\theta,c}{\longmapsto}f(x+\lambda)\end{aligned}$$

Current note has 0 direct children and 0 total descendants.

- [squishy](#)

• [Heisenberg.group](#)

• $H(3,\mathbb{R})=N(3,\mathbb{R})$

• $H(3,\mathbb{R})/\mathbb{Z}$

And it has 1 siblings.

- [squishy](#)

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• $H(3,\mathbb{R})=N(3,\mathbb{R})$

• $H(3,\mathbb{R})/\mathbb{Z}$