

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on January 12, 2026 4:58:49 PM, was last modified on February 7, 2026 8:01:27 PM, and was exported on September 7, 2026 3:24:51 PM.

When is a local homeomorphism between topological spaces a covering map when domain is compact

Proposition: Let X, Y be Hausdorff spaces where X is compact and Y is connected. Then a local homeomorphism

$$f : X \rightarrow Y$$

is a covering map.

Let $f : X \rightarrow Y$ be a local homeomorphism.

- As local homeomorphism is open, we have $f(X)$ is both open and closed (because X is compact and Y is Hausdorff).
 - Thus by connectedness of Y we have $Y = f(X)$.
- As $y \in Y$ is closed, we have $f^{-1}(y)$ is a closed subset of compact $X \implies f^{-1}(y)$ is compact.
 - As f is a local homeomorphism, for every $x \in f^{-1}(y)$ there is a $U_x \subseteq Y$ such that

$$f : U_x \rightarrow f(U_x)$$

is a homeomorphism, in particular a bijection, so $f^{-1}(y) \cap U_x = \{x\}$.

- Thus $f^{-1}(y)$ is compact and discrete so it is finite.
- Let

$$f^{-1}(y) = \{x_1, \dots, x_n\}$$

where x_j are all distinct points of f .

- We have onb U_{x_j} of each x_j such that

$$f : U_{x_j} \rightarrow f(U_{x_j})$$

is a homeomorphism.

- Now consider

$$V_y := \left(\bigcap_{1 \leq i \leq n} \underbrace{f(U_{x_i})}_{\text{open}} \right) \setminus f \left(X \setminus \bigcup_{1 \leq i \leq n} U_{x_i} \right)$$

surjective local homeomorphisms with constant finite fiber cardinality

Proposition: Let X, Y be Hausdorff spaces and

$$f : X \rightarrow Y$$

be a surjective local homeomorphism such that $|f^{-1}(y)| = n \in \mathbb{Z}_{>0}$ for all $y \in Y$ ($\iff$ fibers are compact and have constant cardinality). Then f is a finite covering map.

Let

$$f^{-1}(y) = \{x_1, \dots, x_n\}$$

- Let V_i be a onb of x_i such that $\{V_i\}$ is pairwise disjoint and $f|_{V_i}$ is a homeomorphism onto its image.
- Let

$$U_y := \bigcup_{1 \leq i \leq n} f(V_i)$$

- Then if $y_1 \in U_y := \bigcup_{1 \leq i \leq n} f(V_i)$ we have

$$f^{-1}(y_1) \cap \bigsqcup_{1 \leq i \leq n} V_i = \{v_1, \dots, v_n\}$$

- But as $|f^{-1}(y_1)| = n$ then

$$f^{-1}(y_1) = \{v_1, \dots, v_n\} \subseteq \bigsqcup_{1 \leq i \leq n} V_i$$

for any $y_1 \in U_y$.

- Thus

$$f^{-1}(U_y) \subseteq \bigsqcup_{1 \leq i \leq n} V_i$$

[1]

when the map is proper

[2]

Proposition: Let X, Y be connected, locally path-connected and compactly generated Hausdorff spaces. Then a **proper** local homeomorphism

$$f : X \rightarrow Y$$

($\iff$ closed local homeomorphism with finite fibers) is a covering map.

Current note has 0 direct children and 0 total descendants.

- [structures based on sets](#)
 - [Topological spaces](#)
 - [map](#)
 - [When is a local homeomorphism between topological spaces a covering map](#)

And it has 12 siblings.

- [structures based on sets](#)
 - [Topological spaces](#)
 - [map](#)
 - [Closed map between topological spaces](#)
 - [Continuous maps between topological spaces](#)
 - [When does a map between topological spaces send discrete sets to discrete sets](#)
 - [Homotopy classes of maps](#)
 - [Iterative dynamics of a continuous function on a topological space](#)
 - [Local homeomorphisms between topological spaces](#)
 - [Open maps between topological spaces](#)
 - [Projection modulo compact factor](#)
 - [Proper maps between topological spaces](#)
 - [Quotient maps between topological spaces](#)
 - [When is a local homeomorphism between topological spaces a covering map](#)
 - [When is a local homeomorphism between topological spaces a closed map?](#)

-
1. <https://math.stackexchange.com/a/3967190> ↩
 - 11-9. Show that a proper local homeomorphism between connected, locally path-connected, and compactly generated Hausdorff spaces is a covering map.
 2. 11-10. Show that a covering map is proper if and only if it is finite-sheeted.
- ↩