Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on April 1, 2026 2:19:19 AM,
was last modified on August 28, 2026 1:09:19 PM, and
was exported on September 7, 2026 3:09:48 PM.

Torsion free/finitely generated Kleinian groups

$\leftrightarrow$

oriented hyperbolic 3-manifolds with geometrically finite or simply degenerate ends

$\leftrightarrow$

topologically tame oriented hyperbolic 3-manifolds/finite degree orbifold covers of

$\to ?$

disjoint union of finitely many compact Riemann surfaces minus finitely many points/with branching signature

	$\mathrm{RepDF}(\pi_1(M), PSL(2)(\mathbb{C}))$	**interior**
∂M empty or disjoint union of tori		2 points
handlebody		ultra-Schottky?
$T_g^2 \times [0,1]$		

Quasiconformal deformations of finitely generated Kleinian groups

Teichmuller space of Kleinian groups

[1]

Definition. Teichmuller space of finitely generated Kleinian groups

Let $\Gamma < \mathrm{Isom}(\mathbb{R}\boldsymbol{H}^3)$ be a non-elementary finitely generated Kleinian group whose limit set is connected. The **Teichmuller space** of Γ is

$$\mathcal{T}(\Gamma) := {\left\{\rho \in \mathrm{HomGrp}(\Gamma, \mathrm{Isom}(\mathbb{R}\boldsymbol{H}^3)) \middle| \begin{matrix}\exists \rho\text{-equivariant} \\ f \in \mathrm{qc}^+(\partial \mathbb{R}\boldsymbol{H}^2)\end{matrix}\right\}}\Big/ PSL(2)(\mathbb{C})$$

(Bers identification) From

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and further let Γ be torsion-free. Let

$$\Gamma \backslash \Omega(\Gamma) = \Sigma_1 \sqcup \cdots \sqcup \Sigma_n$$

be disjoint union of connected components and

$$\mathcal{T}(\Sigma) := \mathcal{T}(\Sigma_1) \times \cdots \times \mathcal{T}(\Sigma_n)$$

Then there exists a natural homeomorphism

$$\beta : \mathcal{T}(\Sigma) \cong_{\mathsf{Top}} \mathcal{T}(\Gamma)$$

which is an isometry between Teichmuller metric and maximum of Teichmuller metrics on $\mathcal{T}(\Sigma_k)$.

1. Michael Kapovich - Hyperbolic Manifolds and Discrete Groups, p.190 ↩

Tameness theorems

(Ahlfors finiteness theorem) Let Γ be a non-elementary finitely generated Kleinian group. Then

$$\Gamma\backslash^{\Omega(\Gamma)}$$

is a finite union of Riemann surfaces of finite type.

(Marden's tameness conjecture turned theorem by Agol, Calegari-Gabai) Hyperbolic 3-manifolds with finitely generated fundamental groups are (*topologically tame*) homeomorphic to the interior of a compact 3-manifold possibly with boundary.

[1]

(Ahlfors measure zero theorem) The limit set of a finitely generated Kleinian group Γ is either $\partial\mathbb{R}\boldsymbol{H}^2$ or a of Lebesgue measure 0.

Deformations of a compact hyperbolic 3-manifold possibly with boundary

Definition. Deformations of a compact hyperbolic 3-manifold possibly with boundary

Let M be a hyperbolizable compact 3-manifold possibly with boundary. The space of all *marked, oriented hyperbolic* 3*-manifolds* homotopy equivalent to M/with fundamental group isomorphic to $\pi_1(M,\bullet)$ is

$$\begin{gathered}\mathrm{RepDF}(\pi_1(M,\bullet), PSL(2)(\mathbb{C}))\\ \subset \mathrm{Rep}(\pi_1(M,\bullet), PSL(2)(\mathbb{C}))\end{gathered}$$

equipped with the subspace topology.

Theorem 2.3. (Local to Global Principle) *Given $K \geq 1$, $C \geq 0$, there exists $\hat{K}$, $\hat{C}$ and A so that if J is an interval in $\mathbb{R}$ and $h : J \to \mathbb{H}^3$ is a (K, C)-quasi-isometric embedding restricted to every connected subsegment of J with length $\leq A$, then h is a $(\hat{K}, \hat{C})$-quasi-isometric embedding.*

Theorem 2.2. *If $\rho : \pi_1(M) \to \mathsf{PSL}(2, \mathbb{C})$ is convex cocompact, then there exists a neighborhood U of ρ in $\mathrm{Hom}(\pi_1(M), \mathsf{PSL}(2, \mathbb{C}))$ such that if $\sigma \in U$, then σ is convex cocompact.*

Corollary 2.4. *If M is a compact, irreducible 3-manifold with non-empty boundary, then $CC(M)$ is open in $X(M)$.*

Theorem 2.2. *If $\rho : \pi_1(M) \to \mathsf{PSL}(2, \mathbb{C})$ is convex cocompact, then there exists a neighborhood U of ρ in $\mathrm{Hom}(\pi_1(M), \mathsf{PSL}(2, \mathbb{C}))$ such that if $\sigma \in U$, then σ is convex cocompact.*

Theorem 5.2. *Let M be a compact, oriented, irreducible 3-manifold with non-empty boundary such that $\pi_1(M)$ is infinite, non-abelian and does not contain a free abelian subgroup of rank 2. Then $CC(M)$ is the interior of $AH(M)$.*

Definition. Marked manifolds homeomorphic to M

$$\mathcal{A}(M) := \frac{\{h : M' \to M \text{ homotopy equivalence} | \text{oriented, cpt, irred, atoroidal 3-manifold}\}}{h_1 \sim h_2 \iff \exists \text{orientation-preserving } j : M_1 \cong_{\mathsf{Top}} M_2 : j \circ h_1 \text{ is homotopic to } h_2}$$

(Ahlfors, Bers, Kra, Marden, Maskit, Sullivan, Thurston) In

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equipped with the subspace topology.

we have

$$\mathrm{int}(\mathrm{RepDF}(\pi_1(M), PSL(2)(\mathbb{C}))) \cong \bigsqcup_{M' \in \mathcal{A}(M)} \mathrm{Mod}_0(M')\backslash^{\mathcal{T}(\partial_{\mathrm{NT}} M')}$$

(Bers, Sullivan and Thurston's density conjecture turned Namazi-Souto, Ohshika theorem) In

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equipped with the subspace topology.

we have

$$\overline{\mathrm{int}(\mathrm{RepDF}(\pi_1(M), PSL(2)(\mathbb{C})))} = \mathrm{RepDF}(\pi_1(M), PSL(2)(\mathbb{C}))$$

[2]

[3]

Current note has 1 direct children and 1 total descendants.

- stem
  - Objects stemming from $PSL(2)(\mathbb{C})$
    - Kleinian groups $\Gamma <_{\mathrm{d}} PSL(2)(\mathbb{C})$ $\leftrightarrow$ Hyperbolic 3-manifolds/orbifolds $_{\Gamma}\backslash^{\mathbb{R}\boldsymbol{H}^3}$
      - Finitely generated Kleinian groups $\leftrightarrow$ topologically tame oriented hyperbolic 3-manifolds/finite degree orbifold covers of
        - Finiteness of finitely generated Kleinian groups

And it has 7 siblings.

- stem
  - Objects stemming from $PSL(2)(\mathbb{C})$
    - Kleinian groups $\Gamma <_{\mathrm{d}} PSL(2)(\mathbb{C})$ $\leftrightarrow$ Hyperbolic 3-manifolds/orbifolds $_{\Gamma}\backslash^{\mathbb{R}\boldsymbol{H}^3}$
      - Finitely generated, free Kleinian groups
      - 2-generated Kleinian groups
      - Finitely generated Kleinian groups $\leftrightarrow$ topologically tame oriented hyperbolic 3-manifolds/finite degree orbifold covers of
      - Building hyperbolic 3-manifolds by gluing polyhedra
      - Kleinian group with $\Lambda(\Gamma) \neq \partial\mathbb{R}\boldsymbol{H}^3$
      - Thick-think decomposition of hyperbolic 3-manifolds
      - Kleinian ultra-Schottky groups $\leftrightarrow$ convex cocompact hyperbolic interior of 3-handlebodies

1. https://websites.umich.edu/~canary//tamenessrevise.pdf ↩
2. Richard D. Canary, 2010 - Introductory bumponomics: the topology of deformation spaces of hyperbolic 3-manifolds ↩
3. Richard Canary - Notes on hyperbolic 3-manifolds ↩