Info

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Prime-representing function

Definition. A **prime-representing function** is a map

$$f : \mathbb{N} \to \mathrm{primes}(\mathbb{N})$$

(Difference between consecutive primes) There is an integer $K \in \mathbb{N}$ such that

$$p_{n+1} - p_n < K p_n^{\frac{5}{8}}$$

where p_n is the n-th prime number.

Proposition: For K in

(Difference between consecutive primes) There is an integer $K \in \mathbb{N}$ such that

$$p_{n+1} - p_n < K p_n^{\frac{5}{8}}$$

where p_n is the n-th prime number.

we have $\forall N > K^8, \exists p \in \mathrm{primes}(\mathbb{N})$

$$N^3 < p < (N+1)^3 - 1$$

Let p_n be the greatest prime less than N^3, that is $p_n := \max[0, N^3) \cap \mathrm{primes}(\mathbb{N})$. Then

$$\begin{aligned} N^3 &< p_{n+1} \\ &< p_n + K p_n^{\frac{5}{8}} \\ &< N^3 + K N^{\frac{15}{8}} \\ &< N^3 + N^2 \\ &< N^3 + 3N^2 + 3N \\ &= (N+1)^3 - 1 \end{aligned}$$

Proposition: Let $q_0 > K^8$ be a prime. Then there exists an infinite sequence of primes

$$q_0, q_1, \ldots$$

such that

$$\forall n > 0, q_n^3 < q_{n+1} < (q_n + 1)^3 - 1$$

We have $\forall n > 0$

$$\begin{aligned} (q_n + 1)^{\frac{1}{3^n}} &> (q_n)^{\frac{1}{3^n}} \\ v(n+1) &> (q_n)^{\frac{1}{3^n}} \\ v(n+1) &< (q_n + 1)^{\frac{1}{3^n}} \end{aligned}$$

Therefore, $(q_n + 1)^{\frac{1}{3^n}}$ is decreasing and $(q_n)^{\frac{1}{3^n}}$ is strictly smaller than $(q_n + 1)^{\frac{1}{3^n}}$. Thus, $(q_n)^{\frac{1}{3^n}}$ is bounded and strictly increasing.

- Therefore, $(q_n)^{\frac{1}{3^n}}$ is a convergent sequence and

  $$A := \lim_{n \to \infty} (q_n)^{\frac{1}{3^n}} \in \mathbb{N}$$

  exists.
  - We have $\forall n > 0$

    $$\begin{aligned} (q_n)^{\frac{1}{3^n}} &< A < (q_n + 1)^{\frac{1}{3^n}} \\ \implies q_n &< A^{3^n} < q_n + 1 \end{aligned}$$

(W H Mills' prime-representing function) There exists $A \in \mathbb{R}$ such that

$$\forall n \in \mathbb{N}_{>0}, \lfloor A^{3^n} \rfloor \in \mathrm{primes}(\mathbb{N})$$

[1]
[2]
[3]

Current note has 0 direct children and 0 total descendants.

- stretchy
  - $\mathbb{Z}$
    - Prime-representing function

And it has 6 siblings.

- stretchy
  - $\mathbb{Z}$
    - Division in $\mathbb{Z}$
    - Euler totient function
    - gcd of integers
    - Primes in $\mathbb{N}$
    - Partitions of naturals
    - Prime-representing function

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1. https://www.ams.org/journals/bull/1947-53-06/S0002-9904-1947-08849-2/S0002-9904-1947-08849-2.pdf ↩
2. https://proofwiki.org/wiki/Mills%27_Theorem ↩
3. https://imar.ro/journals/Mathematical_Reports/Pdfs/2024/3_4/4.pdf ↩