Info

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Teichmuller space of surfaces

Definition. Hyperbolic structures on a surface and the Teichmuller space

Let $S := S_{g,n,b}$ with $\chi(S) = 2 - 2g < 0$. A **hyperbolic structure** on S is a diffeomorphism

$$\phi : S \to (X, g)$$

where (X, g) is a complete hyperbolic 2-manifold with totally geodesic boundary.
Two hyperbolic structures $\phi_1 : S \to X_1$ and $\phi_2 : S \to X_2$ are **homotopic** if there is an isometry $\varphi : X_1 \to X_2$ such that $\varphi \circ \phi_1$ and $\varphi \circ \phi_2$ are homotopic (possibly moving points in ∂X_2).
The **Teichmuller space** of S is the set of all homotopy classes of hyperbolic structures on S

$$\mathrm{Teich}(S) := \frac{\{\text{hyperbolic structures on } S\}}{\text{homotopy}}$$

We can also describe the Teichmuller space as *isotopy* classes of hyperbolic metrics on S

$$\mathrm{Teich}(S) \leftrightarrow \frac{\{\text{hyperbolic metrics on } S\}}{\mathrm{Diff}_0(S)}$$

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And it has 4 siblings.

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