Info

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Smooth singular homology of a smooth manifold

Definition. Smooth simplices

A **smooth k-simplex** in a smooth manifold M is a map

$$\sigma : \Delta^k \to M$$

which has a smooth extension on the nbd of every point.

Definition. Smooth singular homology of a smooth manifold

Generate the smooth singular k-chains over a ring R

$$\mathrm{sing}_k^\infty(M, R)$$

from the set of all smooth k-simplices

> Definition. Smooth simplices
>
> A **smooth k-simplex** in a smooth manifold M is a map
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> $$\sigma : \Delta^k \to M$$
>
> which has a smooth extension on the nbd of every point.

The boundary of a smooth k-simplex

$$\partial_k \sigma := \sigma|_{\partial \Delta^k}$$

gives a group homomorphism

$$\partial_k : \mathrm{sing}_k^\infty \to \mathrm{sing}_{k-1}^\infty$$

making

$$(\mathrm{sing}_\bullet^\infty, \partial_\bullet)$$

the smooth singular chain complex of M.

The *smooth singular homology group* is the homology group of the chain complex

$$H_k \mathrm{sing}^\infty(M, R) := \frac{\ker(\partial_k : \mathrm{sing}_k^\infty \to \mathrm{sing}_{k-1}^\infty)}{\mathrm{im}\ (\partial_k : \mathrm{sing}_{k+1}^\infty \to \mathrm{sing}_k^\infty)}$$

- integration

  $$\int : \mathrm{Forms}^{\mathrm{k}}(M) \times \mathrm{sing}_k^\infty(M, \mathbb{R}) \to \mathbb{R}$$

  gives a map?

  $$\int : \mathrm{Forms}^{\mathrm{k}}(M) \to \mathrm{sing}_\infty^k(M, \mathbb{R})$$

- Dual differential forms of smooth singular chains
- $Z_{k-1}\mathrm{sing}^\infty(M, \mathbb{Z}) \to U(1)$ differential characters
- identifications
  - de Rham chain complex and cohomology in a smooth manifold

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Current note has 2 direct children and 2 total descendants.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Smooth singular homology of a smooth manifold
        - $Z_{k-1}\mathrm{sing}^\infty(M, \mathbb{Z}) \to U(1)$ differential characters
        - Dual differential forms of smooth singular chains

And it has 19 siblings.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Smooth Lie group action on a smooth manifold
      - Borel measures on a smooth manifold
      - G-bundles over smooth manifolds $G \curvearrowright P \to X$
      - Vector bundle on smooth manifolds
      - Curves in smooth manifolds
      - Density on smooth manifolds
      - Functions between smooth manifolds
      - Functions from smooth manifolds to $\mathbb{R}$
      - Flows on a smooth manifold
      - Locally $G \curvearrowright X$-manifolds
      - Smooth singular homology of a smooth manifold
      - Isomorphisms and automorphisms of smooth manifolds

- Lagrangian flows
- Local transformation rules
- Moduli space of Riemannian metrics on a smooth manifold
- Moduli space of quotients
- Quotient manifolds
- Submanifolds of smooth manifolds
- Tangents on a smooth manifold