ⓘ Info

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Hyperbolic ball model

Definition. The **Poincare metric** on B^n

$$d_{\mathbb{R}\boldsymbol{H}\text{-B}} : B^n \times B^n \to [0, \infty)$$

is given by

$$\cosh d_{\mathbb{R}\boldsymbol{H}\text{-B}}(x, y) = 1 + 2\frac{|x - y|^2}{(1 - |x|^2)(1 - |y|^2)}$$

> **Proposition:**
>
> $$\exp d_{\mathbb{R}\boldsymbol{H}\text{-B}}(x, y) = 1 + 2A(x, y) + 2\sqrt{A(x, y)(1 + A(x, y))}$$

Busemann function

$$\begin{aligned} B_\xi(x, y) &= \lim_{z \to \xi} \log \frac{1 + 2A(x, z) + 2\sqrt{A(x, z)(1 + A(x, z))}}{1 + 2A(y, z) + 2\sqrt{A(y, z)(1 + A(y, z))}} \\ &= \log \frac{\frac{1}{A(x, r\xi)} + 2 + 2\sqrt{}}{} \end{aligned}$$

$$\log \frac{P(y, \xi)}{P(x, \xi)} = (n - 1)B_\xi(y, x)$$

> **Proposition:**
>
> $$B_\xi(x, y) = \log \left(\frac{1 - |x|^2}{1 - |y|^2} \frac{|y - \xi|^2}{|x - \xi|^2} \right)$$

Poisson kernel

$$\begin{aligned} P(x, \xi) &= \left(\frac{1 - |x|^2}{1 + |x|^2 - 2(x \cdot \xi)} \right)^{n-1} \\ &= \left(\frac{1 - |x|^2}{|x - \xi|^2} \right)^{n-1} \end{aligned}$$

Current note has 0 direct children and 0 total descendants.

- squishy
  - $\mathbb{R}\boldsymbol{H}^n$
    - Hyperbolic ball model

And it has 5 siblings.

- squishy
  - $\mathbb{R}\boldsymbol{H}^n$
    - Hyperbolic ball model
    - Conic neighborhood
    - Hyperboloid model of $\mathbb{R}\boldsymbol{H}^n$
    - Hyperboloid model to ball model of $\mathbb{R}\boldsymbol{H}^n$
    - Types of isometries of $\mathbb{R}$-hyperbolic space