Info

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Convex cocompact discrete subgroups of $\mathrm{Isom}(\mathbb{R}\boldsymbol{H}^n) \cong O^+(1,n)$

Let $\Gamma <_{\mathrm{d}} \mathrm{Isom}(\mathbb{R}\boldsymbol{H}^n)$. Then $\Lambda_c(\Gamma) = \Lambda(\Gamma)$

$$\iff$$

Γ is geometrically finite with no cusps

$$\iff$$

Γ is geometrically finite with no parabolic elements

$$\iff$$

for some $p \in \mathbb{R}\boldsymbol{H}^n$ the orbit map $\Gamma \to \mathbb{R}\boldsymbol{H}^n$ is a quasi-isometric embedding

$$\iff$$

for every $p \in \mathbb{R}\boldsymbol{H}^n$ the orbit map $\Gamma \to \mathbb{R}\boldsymbol{H}^n$ is a quasi-isometric embedding

$$\iff$$

the action $\Gamma \curvearrowright \mathrm{convex}(\Lambda(\Gamma))$ is cocompact.

Lemma 2.2 *Suppose we have a representation $\rho\colon G \to \mathrm{Isom}(\mathbb{H}^n)$ of a finitely-generated group G. Then the action $\rho\colon G \curvearrowright \mathbb{H}^n$ is properly discontinuous and convex-cocompact iff there exists a G–equivariant quasi-isometric embedding $f\colon \Gamma_G \to \mathbb{H}^n$, where Γ_G is a Cayley graph of G.*

Proof First, suppose that $\rho\colon G \curvearrowright \mathbb{H}^n$ properly discontinuous and convex-cocompact. Then, because $C(G)$ is a geodesic metric space, there exists a G–equivariant quasi-isometry $f\colon \Gamma_G \to C(G)$. Composing this map with the isometric embedding $\iota\colon C(G) \to \mathbb{H}^n$, we conclude that $f\colon \Gamma_G \to \mathbb{H}^n$ is a quasi-isometric embedding.

Conversely, suppose that $f\colon \Gamma_G \to \mathbb{H}^n$ is an equivariant quasi-isometric embedding. In particular, f is a proper map. Hence, if for $1 \in \Gamma_G$ we set $o := f(1)$, then for each compact subset $K \subset \mathbb{H}^n$ there are only finitely many elements $g \in G$ such that $g(o) \in K$. Therefore the action $G \curvearrowright \mathbb{H}^n$ is properly discontinuous. In particular, it has finite kernel.

Observe that *stability* of quasi-geodesics in $\mathbb{H}^n$ implies that $\mathrm{Im}(f)$ is *quasi-convex*, ie, there exists a constant $c < \infty$ such that for any two points $x, y \in \mathrm{Im}(f)$ the geodesic segment $\overline{xy}$ is contained in a c–neighborhood $N_c(\mathrm{Im}(f))$ of $\mathrm{Im}(f)$. On the other hand, by [6, Proposition 2.5.4], there exists $R = R(c)$ such that the convex hull of each c–quasi-convex subset $S \subset \mathbb{H}^n$ is contained in the R–neighborhood $N_R(S)$. Thus, the convex subset $C(\rho(G)) \subset \mathbb{H}^n$ is contained in $N_{R(c)}(\mathrm{Im}(f))$. Since G acts cocompactly on $\mathrm{Im}(f)$ it follows that G acts cocompactly on $C(G)$. Therefore $\rho(G)$ is convex-cocompact. □

Current note has 0 direct children and 0 total descendants.

- stem
  - O+ R1 n
    - Discrete subgroups of $O^+(1,n)$ ↔ hyperbolic n-manifolds/orbifolds
      - Convex cocompact discrete subgroups

And it has 11 siblings.

- stem
  - O+ R1 n
    - Discrete subgroups of $O^+(1,n)$ ↔ hyperbolic n-manifolds/orbifolds
      - Convex cocompact discrete subgroups
      - Combination of discrete subgroups of $O^+(1,n)(\mathbb{R})$
      - Exponential sum of a discrete subgroup of $\mathrm{Isom}(\mathbb{R}\boldsymbol{H}^n)$
      - $\mathrm{RepDF}(F_g, O^+(1,n)(\mathbb{R}))$
      - Measure homology on a hyperbolic manifold
      - Discrete subgroups with limit set a discontinuum
      - Limit sets of discrete subgroups
      - Discrete subgroups with limit set a $k-1$-sphere
      - Counting orbits of discrete subgroups of $O^+(1,n)(\mathbb{R})$ in $\mathbb{R}\boldsymbol{H}^n$
      - Rigidity of $\mathbb{R}$-hyperbolic manifolds
      - $\mathrm{RepDF}(\pi_1(T_g^2, \bullet), O^+(1,n)(\mathbb{R}))$