Info

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Integral inequalities

(Holder's inequality) Let $1 < p \leq \infty$ and f, g are measurable functions on $(X, \mathfrak{M}, \mu)$. Then

$$\|fg\|_1 \leq \|f\|_p \|g\|_{p^*}$$

In particular

$$f \in L^p(X, \mu), g \in L^q(X, \mu) \implies fg \in L^1(X, \mu)$$

The equality holds $\iff$ $|f|^p = c|g|^q$ for some $c \in \mathbb{R}$.

(Minkowski's inequality) If $1 \leq p \leq \infty$ and $f, g \in L^p(X, \mu)$ then

$$\|f + g\|_p \leq \|f\|_p + \|g\|_p$$

> **Proposition:** Let (X, μ) be a measure space such that $\mu(X) < \infty$. We have
>
> $$\begin{aligned} 0 < p < q \leq \infty \implies & L^p(X, \mu) \leq L^q(X, \mu) \\ \text{and } & \|f\|_p \leq \|f\|_q \, \mu(X)^{\frac{1}{p} - \frac{1}{q}} \end{aligned}$$
>
> Thus, in particular
>
> - for $\mu(X) = 1$
>
> $$\begin{aligned} L^1 \geq L^2 \geq \cdots \geq L^\infty \\ \| \ \|_1 \leq \| \ \|_2 \leq \cdots \leq \| \ \|_\infty \end{aligned}$$
>
> - the inclusions
>
> $$\iota : (L^p(X, \mu), \| \ \|_p) \hookrightarrow (L^q(X, \mu), \| \ \|_q)$$
>
> are continuous(?) for $0 < p < q < \infty$ and has a dense image.

[1]

	$\int_B \lvert f \rvert = \int_X \lvert f \chi_B \rvert \leq \|f\|_\infty \|\chi_B\|_1 = \|f\|_\infty \mu(B$
$f, 1$	

(Minkowski's integral inequality)

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Measure spaces
    - $[0, \infty]$-measure spaces
      - Integral inequalities

And it has 4 siblings.

- structures based on sets
  - Measure spaces
    - $[0, \infty]$-measure spaces
      - Measurable action of a topological group on a positive measure space
      - Integral inequalities
      - Integration of measurable functions with respect to a $[0, \infty]$-measure
      - Monotonic subsets in a measure space

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1. https://math.stackexchange.com/a/3508247 ↩