Info

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Tangents on smooth manifolds as derivations $\mathcal{C}^\infty(M) \to \mathbb{R}$

- A tangent vector $X_p \in T_pM$ is a $\mathbb{R}$-linear map from $\mathcal{C}^\infty(M)$

  $$X_p : \mathcal{C}^\infty(M) \to \mathbb{R}$$

  which maps functions to its directional derivative

  $$f \mapsto X_p f$$

  which satisfies the property

  $$X_p(fg) = X_p(f)\, g(p) + f(p)\, X_p(g)$$

  making it a derivation $\mathcal{C}^\infty(M) \to \mathbb{R}$ at $p \in M$, hence

  $$T_pM \subseteq \mathrm{Der}_p(\mathcal{C}^\infty(M), \mathbb{R})$$

- Now, given any derivation

  $$\delta : \mathcal{C}^\infty(M) \to \mathbb{R}$$

  we use *Taylor's theorem in* $\mathbb{R}^n$ to have $f \in \mathcal{C}^\infty(M)$ written like

  $$f(y) = f(p) + (x - p)$$

  - Hence, the converse is also true.

$$T_pM = \mathrm{Der}_p(\mathcal{C}^\infty(M), \mathbb{R})$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Tangent space of a smooth manifold
          - Tangents on smooth manifolds as derivations $\mathcal{C}^\infty(M) \to \mathbb{R}$

And it has 8 siblings.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Tangent space of a smooth manifold
          - Canonical bijection of T_pM with equivalence classes of velocities
          - Tangents on smooth manifolds as derivations $\mathcal{C}^\infty(M) \to \mathbb{R}$
          - local basis of a tangent space of a smooth manifold
          - Tangent bundle TM of a smooth manifold M
          - Cotangent bundle T^*M on a smooth manifold M
          - Comparison of constructions with tangent vectors
          - Tangent frames on smooth manifolds
          - Tangent vector fields on a smooth manifold