

Info

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$O(n,1)(\mathbb{R})$

Definition. $O(n,1)(\mathbb{R})$

$$O(n+1)(\mathbb{R}) := \left\{ A \in GL(n+1)(\mathbb{R}) \middle| A^T \begin{bmatrix} -1 & \\ & I \end{bmatrix} A = \begin{bmatrix} -1 & \\ & I \end{bmatrix} \right\}$$

Let the $n+1$ -matrix

$$A = \begin{bmatrix} a & v^T \\ w & B \end{bmatrix}, a \in \mathbb{R}, v, w \in \mathbb{R}^n, B \in \mathbf{Mat}(n)(\mathbb{R})$$

then

$$A^T = \begin{bmatrix} a & w^T \\ v & B^T \end{bmatrix}$$

So

$$\begin{aligned} & \begin{bmatrix} a & w^T \\ v & B^T \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} a & v^T \\ w & B \end{bmatrix} \\ &= \begin{bmatrix} -a^2 + w^T w & -av^T + w^T B \\ -av + B^T w & -vv^T + B^T B \end{bmatrix} = \begin{bmatrix} -1 & \\ & I \end{bmatrix} \end{aligned}$$

Then

$$\begin{aligned} \|w\|^2 &= a^2 - 1 \\ B^T B &= I + vv^T \\ B^T w &= av \end{aligned}$$

Current note has 0 direct children and 0 total descendants.

- [squishy](#)

• $\underline{Q}$

• $O(n,1)(\mathbb{R})$

And it has 2 siblings.

- [squishy](#)

• $\underline{Q}$

• $O(n,1)(\mathbb{R})$

• $O^+(1,n)(\mathbb{R})$