Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on February 6, 2026 12:32:04 AM,
was last modified on February 6, 2026 12:50:24 AM, and
was exported on September 7, 2026 3:25:06 PM.

Cross ratios and Mobius maps between metric spaces

Definition. Cross ratios and Mobius maps between metric spaces

Let (X, d) be a metric space. The **cross-ratio** of four pairwise distinct points $x, y, z, w \in X$ is

$$[x, y, z, w]_d := \frac{d(x,z)d(y,w)}{d(x,w)d(y,z)}$$

A map $f : X \to Y$ between metric spaces $(X, d_X), (Y, d_Y)$ is

- **Mobius** if it preserves cross-ratios

$$\forall x, y, z, w \in X, [x, y, z, w]_{d_X} = [f(x), f(y), f(z), f(w)]_{d_Y}$$

- **quasi-Mobius** if

$$\forall x, y, z, w \in X, [x, y, z, w]_{d_X} = \eta([f(x), f(y), f(z), f(w)]_{d_Y})$$

  for some homeomorphism $\eta : [0, \infty) \to [0, \infty)$.

(Efremovich-Tihomirova) Let X, Y be proper δ-hyperbolic spaces and their boundaries $\partial X, \partial Y$ be equipped with visual metrics. Then every quasi-isometry $X \to Y$ extends to a quasi-Mobius homeomorphism $\partial X \to \partial Y$

$$\mathrm{HomMet}_{\mathsf{preQ}}(X, Y) \to \mathrm{MobQ}(\partial X, \partial Y)$$

(Moreover the distortion function η only depends on δ and the quasi...)

[1]

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Metric spaces
    - Moeb
      - Cross ratios and Mobius maps between metric spaces

And it has 1 siblings.

- structures based on sets
  - Metric spaces
    - Moeb
      - Cross ratios and Mobius maps between metric spaces

---

1. Mostow type rigidity theorems - Mostow.pdf ↩