Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on June 13, 2026 5:48:21 PM,
was last modified on June 13, 2026 5:51:20 PM, and
was exported on September 7, 2026 3:09:50 PM.

Regular hypersurfaces in $\mathbb{R}^{2n}$

Definition. Regular hypersurfaces in $\mathbb{R}^{2n}$

A **f-regular hypersurface** in $\mathbb{R}^{2n}$ for $f \in \mathcal{C}^\infty(\mathbb{R}^{2n})$ is a subset given by

$$f^{-1}(c)$$

for some $c \in \mathbb{R}$ such that

- it is a smooth $2n-1$ dimensional submanifold
- $\mathrm{d}f \neq 0$ on $f^{-1}(c)$

closed orbits on hypersurfaces

Question

Does a compact regular H-hypersurface in $(\mathbb{R}^{2n}, \omega_{\mathrm{st}})$ possess a periodic solution of the Hamiltonian vector field X_H?

Proposition: Let $H, F \in \mathcal{C}^\infty(M)$ and

$$S = H^{-1}(c) = F^{-1}(c')$$

with

$$\mathrm{d}H, \mathrm{d}F \neq 0 \text{ on } S$$

(S is both H, F-regular hypersurface). Then the flows of X_H, X_F are reparameterizations of one another. In particular the the periodic orbits of X_H, X_F are the same.

We have

$$\ker \mathrm{d}F = \ker \mathrm{d}H$$

and

$$\mathrm{d}F = \rho \mathrm{d}H \text{ on } S$$

for $\rho \in \mathcal{C}^\infty(S)$.

- This makes

$$X_F = \rho X_H$$

which implies the two vector fields have same orbits.

Moral

Somehow the hypersurface $S \subset \mathbb{R}^{2n}$ carries a prescribed collection of ω_{st}-orbits!

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Regular hypersurfaces in $\mathbb{R}^{2n}$

And it has 39 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - Open subsets of $\mathbb{R}^n$ with $\mathcal{C}^2$ boundary
    - Complex ODEs in $\mathbb{C}$
    - Circle packing on $\mathbb{R}^2$
    - Circle packing converges to the Riemann biholomorphism
    - Bounded connected open subsets of $\mathbb{C}^n$
    - Connected circular open subsets of $\mathbb{C}^n$
    - Continuous functions on $\mathbb{R}^d$
    - Dyadic cubes
    - Curves
    - Differentiable functions
    - Differential forms on $\mathbb{R}^n$
    - Fourier-Wigner transform
    - Harmonic functions composed with conformal maps
    - Hilbert transform
    - Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
    - Holomorphic subsets of $\mathbb{C}^n$
    - Regular hypersurfaces in $\mathbb{R}^{2n}$
    - Orientable hypersurfaces in $\mathbb{R}^n$
    - Laplace operator on $\mathbb{R}^n$
    - $l^\infty(\mathbb{R})$
    - Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$

- Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
- Local $\mathbb{R}$-algebras
- $l^p(\mathbb{R})$
- Metric density of subsets of $\mathbb{R}^n$
- Monotone functions on $\mathbb{R}$
- Cauchy integral of periodic functions
- Polygons with integer vertices
- Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- Discrete subgroups of $\mathbb{R}^n$
- Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- Ramified germs of smooth and holomorphic functions
- Open subsets of $\mathbb{R}^n$
- Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- Closed subsets of $\mathbb{R}^n$ equipped with smooth functions
- Quasi-analytic smooth functions on $\mathbb{R}$
- Star-shaped subsets of $\mathbb{R}^n$
- ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$