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Meromorphic differentials on Riemann surfaces

Definition. Meromorphic differentials on a Riemann surface

Let $n \in \mathbb{Z}$. A **meromorphic differential** of degree n on a Riemann surface X is a collection of meromorphic functions

$$\{\omega_i : U_i \rightarrow \mathbb{C}P^1\}$$

on a open cover $\{U_i\}$ of X with charts $\{z_i : U_i \rightarrow \mathbb{C}\}$ such that

$$\omega_i = (\omega_j \circ z_j \circ z_i^{-1}) \left(\frac{dz_i}{dz_j} \right)^n \text{ on } U_i \cap U_j$$

The space of all meromorphic differentials of degree n

$$\mathcal{G}_{\mathcal{M}}^n(X)$$

forms a $\mathbb{C}$ -vector space with pointwise operations.

The space of all meromorphic differentials

$$\mathcal{G}_{\mathcal{M}}(X) := \bigoplus_{n \in \mathbb{Z}} \mathcal{G}_{\mathcal{M}}^n(X)$$

forms a commutative $\mathbb{Z}$ -graded algebra with the product

$$\begin{aligned} \mathcal{G}_{\mathcal{M}}^n(X) \times \mathcal{G}_{\mathcal{M}}^m(X) &\rightarrow \mathcal{G}_{\mathcal{M}}^{n+m}(X) \\ (\omega, \omega') &\mapsto \omega\omega' := \{\omega_i\omega'_i : U_i \cap U'_i \rightarrow \mathbb{C}P^1\} \end{aligned}$$

Proposition?

$$\mathcal{G}_{\mathcal{M}}(X) \cong \mathcal{M}(X)[t]$$

Definition. Derivative of a meromorphic function is a meromorphic 1-form

The $\mathbb{C}$ -linear map

$$\begin{aligned} d : \mathcal{M}(X) &\rightarrow \Omega_{\mathcal{M}}^1(X) = \mathcal{G}_{\mathcal{M}}^1(X) \\ f &\mapsto \left\{ \frac{\partial f}{\partial z_i} : U_i \rightarrow \mathbb{C} \right\} \end{aligned}$$

has kernel

$$\ker d = \mathbb{C}$$

[1]

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