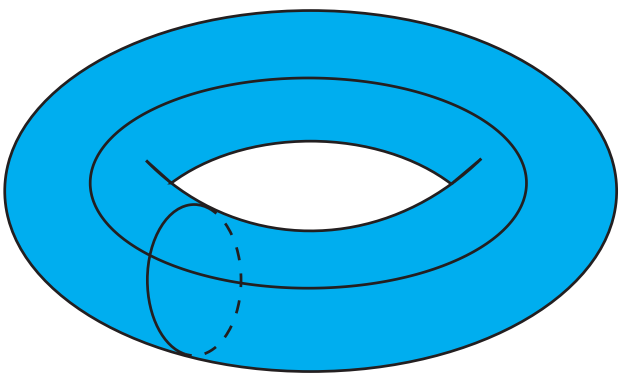
> ⓘ Info
>
> This note, that was found here, as a part of a collection
> is written (completely with human hands) by Rupadarshi Ray,
> was created on July 11, 2023 3:43:46 PM,
> was last modified on September 7, 2026 2:01:33 PM, and
> was exported on September 7, 2026 3:04:04 PM.

2-torus $T^2 := S^1 \times S^1$

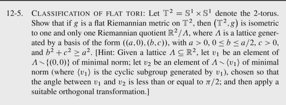
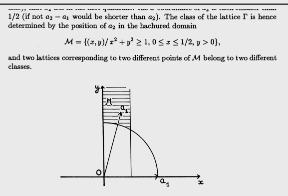
#wiki/Man/R/2

> **Definition.** $T^2 := S^1 \times S^1$
>
> The smooth 2-manifold with a Lie group structure **2-torus** is the product of the Lie group circle S^1 with itself
>
> $$T^2 := S^1 \times S^1$$
>
> 

structures and moduli

> **Proposition**
>
> $$\mathbb{R}^2/\mathbb{Z}^2 \cong_{\text{Lie}} T^2$$

structure	moduli	
isotopy classes of hyperbolic structures	(Teichmuller space) upper-half plane $\mathbb{R}H^2$	
complex structures	quotient space of $SL(2)(\mathbb{Z}) \curvearrowright \mathbb{R}H^2$	set of flat metrics upto conformal

quotients of $L := {}^{GL(2)(\mathbb{R})}/_{GL(2}$	**homeo**	**moduli of lattices/flat tori**	**$\mathbb{Z}$-basis**
L	4-manifold?	all lattices/tori	
$\frac{L}{O_+(2,\mathbb{R})}$	$\mathbb{R}^3$	upto oriented isometries	
$\frac{L}{CO_+(2,\mathbb{R})} \cong ? \frac{H}{SL(2)(}$	$\mathbb{C} \cong \mathbb{R}^2$	upto oriented conformal maps	$v_1 = (1,0)$ and $v_2 \in F$
$\frac{L}{O(2,\mathbb{R})}$	$[0,\infty) \times \mathbb{R}^2$?	upto isometries	 12-5. Classification of flat tori: Let $\mathbb{T}^2 = \mathbb{S}^1 \times \mathbb{S}^1$ denote the 2-torus. Show that if g is a flat Riemannian metric on $\mathbb{T}^2$, then $(\mathbb{T}^2, g)$ is isometric to one and only one Riemannian quotient $\mathbb{R}^2/\Lambda$, where Λ is a lattice generated by a basis of the form $((a,0),(b,c))$, with $a > 0$, $0 \le b \le a/2$, $c > 0$, and $b^2 + c^2 \ge a^2$. [Hint: Given a lattice $\Lambda \subseteq \mathbb{R}^2$, let v_1 be an element of $\Lambda \smallsetminus \{(0,0)\}$ of minimal norm; let v_2 be an element of $\Lambda \smallsetminus \langle v_1 \rangle$ of minimal norm (where $\langle v_1 \rangle$ is the cyclic subgroup generated by v_1), chosen so that the angle between v_1 and v_2 is less than or equal to $\pi/2$; and then apply a suitable orthogonal transformation.] [1]
$\frac{L}{CO(2,\mathbb{R})} \cong ? \frac{H/SL(2}{\mathbb{Z}_2}$	$[0,\infty) \times \mathbb{R}$?	upto (unoriented) conformal maps	 1/2 (if not $a_2 - a_1$ would be shorter than a_2). The class of the lattice Γ is hence determined by the position of a_2 in the hachured domain $\mathcal{M} = \{(x,y)/\ x^2 + y^2 \ge 1,\ 0 \le x \le 1/2,\ y > 0\}$, and two lattices corresponding to two different points of $\mathcal{M}$ belong to two different classes.

1. https://www.desmos.com/calculator/l6fgnougt6 ↩

- normal form: Torus identified with a glued rectangle
- embeddings
  - $$T^2 \hookrightarrow \mathbb{R}^3$$
  - $$T^2 \hookrightarrow \mathbb{R}^4$$

Current note has 2 direct children and 2 total descendants.

- stretchy
  - T^2
    - Torus identified with a glued rectangle
    - Translation flows on the 2-torus

And it has 62 siblings.

- stretchy
  - Trefoil 3_1 knot
  - The only compact connected Lie group that can act faithfully on a torus is itself
  - Hecke algebras
  - $\overline{B^n}$
  - $\mathrm{BS}(1,2)$
  - $(\mathbb{C},+,\times)$
  - $D^* := D \setminus \{0\}$
  - Cantor set
  - Configuration space of N rigid points in $\mathbb{R}^3$
  - Free groups
  - $GL(2)(\mathbb{Z}) \curvearrowright T^2$
  - H_2^3
  - $[a,b]$
  - Figure 8 knot
  - Lamplighter group on $\mathbb{Z}_2$
  - $\beta\mathbb{N}$
  - $\#_2\mathbb{R}P^2$
  - $\prod_{i\in\mathbb{N}}\{1,\ldots,n_i\}$
  - $\mathbb{R}P^n$
  - $(\mathbb{Q},+,\times,<)$
  - $\mathbb{Q}_{\hat{p}}$
  - $\overline{\mathbb{Q}}$
  - $\mathbb{Q}_{\mathrm{constr}}$
  - $\mathbb{Q}(\xi_n)$
  - $\mathcal{O}_{\overline{\mathbb{Q}}}$
  - $(\mathbb{R},+,\times,<,\sup)$
  - Homeomorphism of $\mathbb{R}^2$ that sends $\mathbb{N}$ to a countable closed discrete subset
  - $\mathbb{R}^2 \times S^1$
  - $\mathbb{R}^n$
  - $\mathbb{R} \times S^1$
  - $\mathbb{R} \times S^2$
  - S^1
  - $S^1 \wedge S^1$
  - $S^1 \times S^n$
  - S^2 and $\mathbb{C}\boldsymbol{P}^1$
  - Mapping torus of the antipodal map of S^2
  - S^3
  - S^n
  - I-bundle
  - Symmetric group
  - $\mathbb{T}$
  - T^2
  - T_g^2
  - $T_{0,0,3}^2$
  - $\mathbb{R}^2 \setminus \{0\}$
  - Three $T_{1,0,1}^2$ glued at their boundaries
  - $T_{1,1,0}^2$
  - T_2^2
  - $T_g^2 \times [0,1]$
  - $\#_{\mathbb{N}}T^2$
  - Thompson's group
  - T^n
  - 3-valent tree
  - 4-valent tree
  - $[0,1]^{\mathbb{N}}$ with product topology
  - Whitehead manifolds
  - $\mathbb{Z}$
  - $\mathbb{Z}[\sqrt{m}]$
  - $\mathbb{Z}[X]$
  - $\mathbb{Z}^{\mathbb{N}}$
  - $\mathbb{Z}[i]$
  - $\mathbb{Z}[i\sqrt{5}]$