

Info

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Infinite limit of derivatives

if infinite limits of f, f' exist then $f' \rightarrow 0$

Intuition

If f is a $\mathcal{C}^1[a, \infty)$ curve on $\mathbb{R}$ such that it reaches $f(x) \xrightarrow{x \rightarrow \infty} f(\infty)$ then it should be the case that f goes *slower* and *slower* as $x \rightarrow \infty$. So $f'(x) \xrightarrow{x \rightarrow \infty} 0$.

This however this is not true as if limit of $f'(x)$ does not exist as $x \rightarrow \infty$

Examples: functions on $[1, \infty)$ whose $x \rightarrow \infty$ limit is 0 but their derivatives oscillate or blows up.

	its derivative
$\frac{\cos x}{x}$	
$\frac{\cos x^2}{x}$	
$\frac{\cos x^3}{x}$	$\bullet \frac{d}{dx} \frac{\cos x^3}{x} = -3x \sin(x^3) - \frac{\cos(x^3)}{x^2}$

However, if $f'(x) \xrightarrow{x \rightarrow \infty} f'(\infty)$ exists then the intuitive claim is right!

Suppose for $a \in \mathbb{R}$ we have continuous $f, f' : [a, \infty) \rightarrow \mathbb{R}$ such that the limits

$$\lim_{x \rightarrow \infty} f(x), \lim_{x \rightarrow \infty} f'(x) \in \mathbb{R}$$

exist. Then

$$\lim_{x \rightarrow \infty} f'(x) = 0$$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{e^x f(x)}{e^x} = \lim_{x \rightarrow \infty} (f(x) + f'(x))$$

For example:

- When γ

$$\gamma' = F(\gamma)$$

is **solution of a autonomous continuous vector field** $F : [a, \infty) \rightarrow \mathbb{R}$ such that γ has a $t \rightarrow \infty$ limit $\gamma(\infty) \in [a, \infty)$, then we also have

$$\gamma'(t) = F(\gamma(t)) \xrightarrow{t \rightarrow \infty} F(\gamma(\infty))$$

limit,

- this is where the [above theorem](#) implies

$$\gamma'(\infty) = F(\gamma(\infty)) = 0$$

- So the (future) *endpoint* of the solution (if it exists, when it is a future-global solution) has to be a *fixed point* of the autonomous system!

Current note has 0 direct children and 0 total descendants.

- stem
 - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$
 - Differentiable functions
 - Infinite limit of derivatives

And it has 10 siblings.

- stem
 - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$
 - Differentiable functions
 - Functions $(a, b) \rightarrow \mathbb{R}$ differentiable at a point
 - Comparing a function and it derivative
 - Absolutely continuous functions on $[a, b]$ $\leftrightarrow \int_{[a, -]} (L^1[a, b])$
 - Distributional derivatives
 - Double derivative/Laplace operator on the circle
 - Fractional derivative
 - Infinite limit of derivatives
 - Space of continuous and continuously differentiable functions on $\mathbb{R}$
 - Derivative of maps $\mathbb{R}^n \rightarrow \mathbb{R}^m$
 - Zooming of a map $\mathbb{R}^n \rightarrow \mathbb{R}^m$