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Operator norm of linear maps between $\mathbb{R}^n$

Definition. Operator norm of linear maps between $\mathbb{R}^n$

Let $A \in \text{HomVec}_{\mathbb{R}}(\mathbb{R}^n, \mathbb{R}^m)$ we have

$$\|A\| := \sup\{\|Ax\| \mid x \in \mathbb{R}^n, \|x\| \leq 1\}$$

- Any $x \in \mathbb{R}^n$ we have $x = \sum c_i e_i, \forall i |c_i| \leq \|x\| \leq 1$ then

$$\|Ax\| \leq \sum |c_i| \|Ae_i\| \leq \sum \|Ae_i\|$$

- $\|Ax\| \leq \|A\| \|x\|$

- $\forall x \in \mathbb{R}^n (\|Ax\| \leq \lambda \|x\|) \implies \|A\| \leq \lambda$

- $\|A + B\| \leq \sup_{\|x\| \leq 1} \|(A + B)(x)\| \leq \sup_{\|x\| \leq 1} (\|Ax\| + \|Bx\|) \leq \sup_{\|x\| \leq 1} \|Ax\| + \sup_{\|x\| \leq 1} \|Bx\| \leq \|A\| + \|B\|$



$\text{HomVec}_{\mathbb{R}}(\mathbb{R}^n, \mathbb{R}^m)$

is a normed $\mathbb{R}$ -vector space with the

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And because it is the preimage of the set $\mathbb{R} \setminus \{0\}$ with the map $\det : \text{HomVec}_{\mathbb{R}}(\mathbb{R}^n, \mathbb{R}^m) \rightarrow \mathbb{R}$:



$GL_{\mathbb{R}}(\mathbb{R}^n)$

is an open subset of $\text{HomVec}_{\mathbb{R}}(\mathbb{R}^n, \mathbb{R}^m)$ with the [operator norm](#) and

$$()^{-1} : GL_{\mathbb{R}}(\mathbb{R}^n) \rightarrow GL_{\mathbb{R}}(\mathbb{R}^n)$$

is a continuous map.

- $A \in GL(\mathbb{R}^n), \|B - A\| \|A^{-1}\| < 1 \implies B \in GL(\mathbb{R}^n)$
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- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \setminus \mathbb{R}^n, \Gamma \setminus \mathbb{C}^n\$](#)
 - [mapping](#)
 - [Operator norm of linear maps between \$\mathbb{R}^n\$](#)

And it has 5 siblings.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \setminus \mathbb{R}^n, \Gamma \setminus \mathbb{C}^n\$](#)
 - [mapping](#)
 - [Image of balls under differentiable maps](#)
 - [Inverse of differentiable maps](#)
 - [Mean value property of functions](#)
 - [Operator norm of linear maps between \$\mathbb{R}^n\$](#)
 - [Sequence of functions on \$\mathbb{R}^d\$](#)