ⓘ Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
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$\sum_{n\geq 1} z^{n!}$

[1]

$$z + z^{2!} + z^{3!} + \dots$$

- On $K(\text{compact}) \subset D$, $|z| = r < 1$, then

$$\left|\sum_{n\geq 1} z^{n!}\right| \leq \sum_{n\geq 0} r^n = \frac{1}{1-r} < \infty$$

- Given a rational $\frac{p}{q}, q \geq 1$ [2] [3] for $z = r\exp\left(i\frac{\pi p}{q}\right)$ we have

$$\sum_{n\geq 1}\left(r\exp\left(i\pi\frac{p}{q}\right)\right)^{n!} = \sum_{n\geq 1} r^{n!}\exp\left(i\pi n!\frac{p}{q}\right)$$

  - ...

> **Proposition:** The function
>
> $$\begin{aligned} D &\to \mathbb{C} \\ z &\mapsto \sum_{n\geq 0} z^{n!} \end{aligned}$$
>
> has the unit circle S^1 as its circle of convergence. It is unbounded in every neighborhood of every point on its circle of convergence, thus the unit circle is the natural boundary of the function.

$g(z) = \sum_{n\geq 0} \frac{z^{n!}}{n!}$

$$zg'(z) = \sum_{n\geq 1} z^{n!}$$

Current note has 0 direct children and 0 total descendants.

- squishy
  - pow
    - $\sum_{n\geq 0} z^{n!}$

And it has 28 siblings.

- squishy
  - pow
    - Angle function
    - Bessel functions
    - $$\cos\sqrt{z}$$
    - The complex logarithm
    - $\sum_{n\geq 0} z^{n!}$
    - $z^3 - 3z$
    - $$\frac{r_0}{1+\epsilon\cos b\theta}$$
    - $\cos\left(\pi\frac{p}{q}\right)$
    - $\frac{\cos x^k}{x}$
    - Elliptic integrals and Jacobi elliptic functions
    - The complex exponential $\exp(z)$
    - $\exp\left(\frac{z+1}{z-1}\right)$
    - e^{-x^2}
    - [Only sum terms that are multiples of N in $\mathbb{C}[[z]](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.$
      ]]
    - $\int\frac{1}{x^2+bx+c}$
    - $\frac{\exp(z)}{1-z}$
    - $\sin x$
    - $\sin(x\sin(x))$
    - $\frac{\sin x}{x}$
    - stock.sin x by xk
    - $\sin x^2$
    - $\sqrt{z}$
    - $\sqrt{z^2-1}$
    - Theta functions
    - Weierstrass elliptic function $\wp$
    - Weierstrass' primary factors
    - $x^2\sin\frac{1}{x}$
    - $x + x^2\sin\frac{1}{x}$

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1. math.ualberta.ca/~isaac/math311/s14/lac.pdf ↩
2. https://math.stackexchange.com/a/2156862 ↩

3. https://math.stackexchange.com/a/4985000 ↩