Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on June 28, 2024 11:06:20 PM,
was last modified on July 22, 2025 12:20:06 AM, and
was exported on September 7, 2026 3:31:19 PM.

# Lorentz flows from a 2-form

**Definition.** Lorentz flows

$$(M, g, F)$$

Let $(M, g)$ be a semi-Riemannian manifold and

$$F \in \Omega^2(M), \mathrm{d}F = 0$$

then by the symplectic structure on $T^*M$

$$\omega^{qF} := \mathrm{d}\theta + q\pi^*_{T^*M}F$$

we have the $\omega^{qF}$-Hamiltonian vector field of the smooth function

$$\begin{aligned} T^*M &\to \mathbb{R} \\ p &\mapsto \frac{\|p\|_g^2}{2} \end{aligned}$$

- The equation on $M$ is

$$\nabla_{\dot{x}}\dot{x} = q(\iota_{\dot{x}}F)^\sharp$$

- This means

$$\langle \dot{x}, \nabla_{\dot{x}}\dot{x} \rangle_g = qF(\dot{x}, \dot{x}) = 0$$

  so the acceleration is "perpendicular" to the velocity.

This generalizes

- On $U \subseteq \mathbb{R}^3$ with Euclidean metric, for

$$qB \in \Omega^2(U)$$

  the equation becomes

$$\ddot{x} = \dot{x} \times q \begin{bmatrix} B_{23} \\ B_{31} \\ B_{12} \end{bmatrix}$$

  - On $T^*\mathbb{R}^3 \times S^2$ for

$$qB + s\Omega$$

    gives equations of a charged particle [1]
- On $\mathbb{R}^4$ with Minkowski metric, electromagnetic field $F$, the equation is **Lorentz force equation**.

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- as a Lagrangian A lagrangian with a 2-form
- as a geodesic flow in a higher dimensional space Kaluza-Klein geodesic flow from Lorentz force flow
- as a gauge theory

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Current note has 2 direct children and 2 total descendants.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold semi-Riemannian
      - Vector fields on Riemannian manifolds
        - Lorentz flows from a 2-form
          - Kaluza-Klein geodesic flow from Lorentz force flow
          - A lagrangian with a 2-form

And it has 2 siblings.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold semi-Riemannian
      - Vector fields on Riemannian manifolds
        - Lorentz flows from a 2-form
        - Potential flows

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1. Victor Guillemin, Shlomo Sternberg - Symplectic techniques in physics-Cambridge University Press (1984).pdf > page=146 ↩