Info

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Holomorphic subsets of $\mathbb{C}^n$

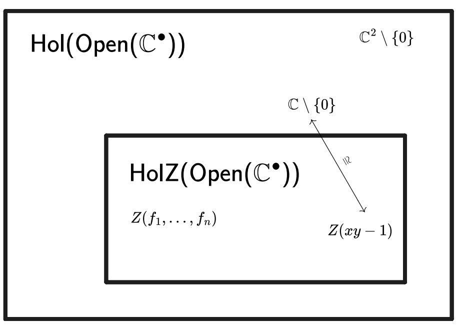
Definition. Definition

Let $U \subseteq \mathbb{C}^n$ be an open subset. A **holomorphic zero set** of U is a zero set of finitely many holomorphic functions $f_1, \dots f_n \in \mathscr{O}(U)$

$$Z(f_1, \dots, f_n) := \{z \in U | f_1(z) = f_2(z) = \cdots = f_n(z) = 0\} = \bigcap_i Z(f_i)$$

A **holomorphic subvariety** of U is a subset $X \subseteq U$ such that for each $p \in X$ there is a nb V_p in U such that $V_p \cap X$ is a holomorphic subset of U, that is,

$$\exists f_1, \dots f_n \in \mathscr{O}(V_p), V_p \cap X = Z(f_1, \dots, f_n)$$



> Let $X \subseteq U(\text{open}) \subseteq \mathbb{C}^n$ be a holomorphic subvariety.

Definition. Germs of holomorphic subvarieties

Let $X_1, X_2 \subseteq U$ be two holomorphic subvarieties of $U \subseteq \mathbb{C}^n$. Then X_1 and X_2 are **equivalent** at $p \in X_1 \cap X_2$ if there is a nb $U_p \subseteq U$ of p such that

$$X_1 \cap U_p = X_2 \cap U_p$$

Therefore, we have the surjection from the set $\mathsf{Hol}(U)$ of all holomorphic subvarieties of U onto the same quotiented out by *equivalence at p*

$$\mathsf{Hol}(U) \twoheadrightarrow \mathsf{Hol}(U)_p$$

A **germ** of holomorphic subvarieties of U at p is an element of $\mathsf{Hol}(U)_p$.

Definition. Holomorphic functions on a subvariety

> Let $X \subseteq U(\text{open}) \subseteq \mathbb{C}^n$ be a holomorphic subvariety.

Then a function

$$f : X \to \mathbb{C}$$

is **holomorphic** if for each $p \in X$ there is a onb $U_p \subseteq \mathbb{C}^n$ such that f extends to $f \in \mathscr{O}(U_p)$.

Let $\mathbb{C}$-algebra of holomorphic functions on X be the set of all holomorphic functions on X

$$\mathscr{O}(X)$$

with pointwise operations and the sheaf of holomorphic functions on X be

$$\begin{aligned} \mathscr{O}_X : \mathsf{Open}(X) &\to \mathsf{Alg}_{\mathbb{C}} \\ V &\mapsto \mathscr{O}_X(V) := \mathscr{O}(V \cap X) \end{aligned}$$

The stalk of the sheaf $\mathscr{O}_X$ at $p \in X$, that is, $\mathbb{C}$-algebra of germs of holomorphic functions

$$\mathscr{O}_{X,p}$$

is the **local ring** of X at p.

irreducible subvarieties

Definition. Definition

> Let $X \subseteq U(\text{open}) \subseteq \mathbb{C}^n$ be a holomorphic subvariety.

Then X is **reducible** if $X = X_1 \cup X_2$ where X_1, X_2 are holomorphic subvarieties of U.
If X is nor reducible, then it is **irreducible**.

[1]

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic subsets of $\mathbb{C}^n$

And it has 39 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, {}_\Gamma\backslash^{\mathbb{C}^n}$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
    - Open subsets of $\mathbb{R}^n$ with $\mathcal{C}^2$ boundary

- Complex ODEs in $\mathbb{C}$
- Circle packing on $\mathbb{R}^2$
- Circle packing converges to the Riemann biholomorphism
- Bounded connected open subsets of $\mathbb{C}^n$
- Connected circular open subsets of $\mathbb{C}^n$
- Continuous functions on $\mathbb{R}^d$
- Dyadic cubes
- Curves
- Differentiable functions
- Differential forms on $\mathbb{R}^n$
- Fourier-Wigner transform
- Harmonic functions composed with conformal maps
- Hilbert transform
- Fourier-Cauchy-Poisson correspondence of holomorphic and harmonic functions on the unit disk and their boundary values
- Holomorphic subsets of $\mathbb{C}^n$
- Regular hypersurfaces in $\mathbb{R}^{2n}$
- Orientable hypersurfaces in $\mathbb{R}^n$
- Laplace operator on $\mathbb{R}^n$
- $l^\infty(\mathbb{R})$
- Lebesgue measurable subsets of and functions on $\mathbb{R}^n, T^n, S^n$
- Lebesgue measure of boundary of open sets in $\mathbb{R}^n$
- Local $\mathbb{R}$-algebras
- $l^p(\mathbb{R})$
- Metric density of subsets of $\mathbb{R}^n$
- Monotone functions on $\mathbb{R}$
- Cauchy integral of periodic functions
- Polygons with integer vertices
- Open smooth maps $U \subseteq \mathbb{R}^2 \to \mathbb{C}$
- Discrete subgroups of $\mathbb{R}^n$
- Discrete cocompact subgroups of $\mathbb{R}^n$, flat tori
- Ramified germs of smooth and holomorphic functions
- Open subsets of $\mathbb{R}^n$
- Open subsets of $\mathbb{R}^n$ equipped with the flat metric
- Closed subsets of $\mathbb{R}^n$ equipped with smooth functions
- Quasi-analytic smooth functions on $\mathbb{R}$
- Star-shaped subsets of $\mathbb{R}^n$
- ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$

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1. Joseph L Taylor - Several Complex Variables with Connections to Algebraic Geometry and Lie Groups (2002), p.46 ↩