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Functions from smooth manifolds to $\mathbb{R}$

Definition. Smoothness of functions $f : M \rightarrow \mathbb{R}$

A function $f : M \rightarrow \mathbb{R}$ is **smooth** if

$$f \circ \varphi^{-1} : \varphi(U) \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$$

is *smooth* in the conventional sense, for a given chart (U, φ) .

$$\mathcal{C}^\infty(M) := \{f : M \rightarrow \mathbb{R} : f \text{ is smooth}\}$$

Smoothness of a function $f : M \rightarrow \mathbb{R}$ by

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does not depend on chart.

derivatives

directional derivatives

Equation

$$X_p(fg) = X_p(f) g(p) + f(p) X_p(g)$$

derivations

Equation

$$T_p M = \text{Der}_p(\mathcal{C}^\infty(M), \mathbb{R})$$

push-forward

$$\mathfrak{D}_p f : T_p M \rightarrow \mathbb{R}$$

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