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Totally geodesic submanifolds of semi-Riemannian manifolds

Definition. Totally geodesic submanifolds of semi-Riemannian manifolds

A Riemannian (immersed/embedded) submanifold

$$\iota : (M, \mathrm{g}_M) \hookrightarrow (X, \mathrm{g}_X)$$

of a semi-Riemannian manifold (X, g_X) is **totally geodesic** if for every g_X-geodesic that is tangent to M at some $t_0 \in \mathbb{R}$ stays in M for all $t \in (t_0 - \epsilon, t_0 + \epsilon)$.

Proposition:
Let

$$\iota : (M, \iota^* \mathrm{g}) \hookrightarrow (X, \mathrm{g})$$

be an *embedded* Riemannian submanifold. Then the following is equivalent

- M is totally geodesic

  Definition. Totally geodesic submanifolds of semi-Riemannian manifolds

  A Riemannian (immersed/embedded) submanifold

  $$\iota : (M, \mathrm{g}_M) \hookrightarrow (X, \mathrm{g}_X)$$

  of a semi-Riemannian manifold (X, g_X) is **totally geodesic** if for every g_X-geodesic that is tangent to M at some $t_0 \in \mathbb{R}$ stays in M for all $t \in (t_0 - \epsilon, t_0 + \epsilon)$.
- Every $\iota^*\mathrm{g}$-geodesic $\gamma : (a, b) \to M$ is also a g-geodesic $\iota \circ \gamma : (a, b) \to X$
- The second fundamental form of M vanishes identically.

[1]

[2]

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold semi-Riemannian
      - Totally geodesic submanifolds of semi-Riemannian manifolds

And it has 18 siblings.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold semi-Riemannian
      - Universal cover of semi-Riemannian manifolds
      - Curvature of a Riemannian manifold
      - Einstein manifolds
      - Exponential map from tangent bundle
      - Geodesics past conjugate points in a Riemannian manifold
      - Riemannian manifolds with ergodic geodesic flow
      - Isometries of a semi-Riemannian manifold
      - Laplacian on a Riemannian manifold
      - Parallel transport on a curve in a semi-Riemannian manifold
      - Conjugate points in a semi-Riemannian manifold
      - Quotients of Riemannian manifolds
      - Complete Riemannian manifolds
      - Riemannian connection
      - Riemannian submanifolds
      - Totally geodesic submanifolds of semi-Riemannian manifolds
      - Tangent bundle of a Riemannian manifold
      - Unit tangent bundle of a Riemannian manifold
      - Vector fields on Riemannian manifolds

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1. **Proposition 8.12.** *Suppose (M, g) is an embedded Riemannian submanifold of a Riemannian or pseudo-Riemannian manifold $(\widetilde{M}, \widetilde{g})$. The following are equivalent:*

   (a) M is totally geodesic in $\widetilde{M}$.
   (b) Every g-geodesic in M is also a $\widetilde{g}$-geodesic in $\widetilde{M}$.
   (c) The second fundamental form of M vanishes identically.

   ↩
2. alberto-2024.pdf ↩