Info

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Totally bounded subsets of a metric space

Definition. Totally bounded subsets of a metric space

A subset $K \subseteq X$ of a metric space X is **totally bounded** if for every $\epsilon > 0$, K can be covered by finitely many ϵ-balls centered inside K.

- If $K \subseteq X$ is compact
  - for every ϵ, take the open cover of K

$$\{B(\epsilon, p) | p \in K\}$$

  - then this must have a finite subcover

$$\{B(\epsilon, p_i) | p_1, \ldots, p_n \in K\}$$

  - thus K is totally bounded.

A subset of a *complete* metric space is compact

$$\iff$$

the subset is **closed and totally bounded**

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