ⓘ Info

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$[0, \infty]$-measure spaces

Definition. $[0, \infty]$-measure spaces $(M, \mathfrak{M}_M, \mu)$

A positive or $[0, \infty]$-**measure space** is a triple $(M, \mathfrak{M}_M, \mu)$ where

- M is a set
- $\mathfrak{M}$ is a σ-algebra on 2^M
- a function called the **measure**

$$\mu : \mathfrak{M}_M \to [0, \infty]$$

  such that
  - $\mu(\emptyset) = 0$
  - *(countable additivity)*

$$\mu\left(\bigcup_{n\geq 1} A_n\right) = \sum_{n\geq 1} \mu(A_n)$$

    where $\{A_n\}_{n\geq 1}$ is a countable collection of pairwise disjoint sets
  - there is at least one $A \in \mathfrak{M}$ such that $\mu(A) < \infty$

- **Definition.** Meas
  - Obj: measure spaces $(M, \mathcal{A}_M, \mu)$
  - Mor: $\mathsf{Hom}_{\mathsf{Meas}}((M, \mathcal{A}_M, \mu), (N, \mathcal{A}_N, \delta))$ are measure preserving maps
- $$L^2 : \mathsf{Meas} \to ?\mathsf{Hilb}$$

Current note has 4 direct children and 4 total descendants.

- structures based on sets
  - Measure spaces
    - $[0, \infty]$-measure spaces
      - Measurable action of a topological group on a positive measure space
      - Integral inequalities
      - Integration of measurable functions with respect to a $[0, \infty]$-measure
      - Monotonic subsets in a measure space

And it has 7 siblings.

- structures based on sets
  - Measure spaces
    - Algebra of subsets of a set
    - Meas
    - Functions on measure spaces
    - $\mathbb{C}$-measures
    - $[0, \infty]$-measure spaces
    - Measure preserving endomorphisms
    - $[0, 1]$-measure spaces