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From the symmetries of a n -gon

Under any symmetry, say the vertex

$$1 \mapsto i$$

then

$$2 \mapsto i - 1 \text{ or } i + 1$$

but after we know what 1, 2 maps to, we know the map completely, because 3 maps to the element which is next to $f(2)$, which only has one choice because $f(1)$ is already at one of the positions.

So, we have

$$(n)(2)$$

choices for $f(1), f(2)$.

Thus we conclude, that D_{2n} contains these rotations

$$i, r, r^2, \dots, r^n - 1$$

where r is the rotation by angle $2\pi/n$. And now let s be the reflction through the line of symmetry passing through 1 and centre of the n -gon.

And now by the definition of r, s the product

$$srs = r^{-1}$$

This gives us

$$\begin{aligned} sr^is &= sr \dots rs \\ &= (srs)(srs) \dots \\ &= r^{-1} \dots \\ &= r^{-i} \end{aligned}$$

This means we have the following

$$s, sr, sr^2, \dots, sr^{n-1}$$

each of them are reflections because

$$(sr^i)^2 = (sr^is)r^i = r^{-i}r^i = i$$

Also, a product of a reflection and a rotation

$$r^i sr^j = r^{i-1}(r^{-1})r^{j-1}$$

is a rotation!

Current note has 0 direct children and 0 total descendants.

- stretchy
 - dihedral
 - D_n
 - From the symmetries of a n -gon

And it has 2 siblings.

- stretchy
 - dihedral
 - D_n
 - Center of the dihedral group
 - From the symmetries of a n -gon