

Info

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on October 16, 2023 2:21:23 PM, was last modified on June 3, 2025 12:24:01 PM, and was exported on September 7, 2026 3:04:03 PM.

Partitions of naturals

Definition. Partitions of naturals

A **partition of** $n \in \mathbb{N}$ is a finite sequence

$$1 \leq n_1 \leq \dots \leq n_k \leq n$$

such that

$$\sum_{i=1}^k n_k = n$$

The number of paritions of n is denoted by $\text{part}(n)$.

$n \in \mathbb{N}$	$\text{part}(n)$
0	1
1	1
2	2
3	3
4	5
5	7
6	11
7	15
8	22
9	30
10	42
11	56
12	77
13	101
14	135
15	176

[1]

Proposition: Ramanujan identities

- $\text{part}(5n + 4) \equiv 0 \bmod 5$
- $\text{part}(7n + 5) \equiv 0 \bmod 7$
- $\text{part}(11n + 6) \equiv 0 \bmod 11$

Conjecture: For each r, m where $0 \leq r < m \exists$ infinitely many n such that

$$\text{part}(n) \equiv r \bmod m$$

- proved for all primes ≥ 5 [2]
- K Ono gave a criteria for verifying the conjec [3]

generating function



$$\sum_{n=0}^{\infty} p(n)x^n = \prod_{n=1}^{\infty} \frac{1}{(1-x^n)}$$

compute partitions



(Euler's Pentagonal theorem)

$$\prod_{n=1}^{\infty} (1-x^n) = \sum_{k=-\infty}^{\infty} (-1)^k x^{k(3k-1)/2}$$

Combine with the generating function get the recurrence relation



$$p(n) = \sum_{k \neq 0} (-1)^{k-1} p(n - g_k)$$

where $g_k := \frac{k(3k-1)}{2}$

which gives

$$p(n) = p(n-1) + p(n-2) - p(n-5) - p(n-7) + p(n-12) + p(n-15) + \dots$$

Because $n < 0 \implies p(n) = 0$, if we start with $p(1)$ and $p(2)$

```
def pentagonal_number(k):
    return int(k*(3*k-1) / 2)

def compute_partitions(goal):
    partitions = [1]
    for n in range(1,goal+1):
```

```

    partitions.append(0)
    for k in range(1,n+1):
        coeff = (-1)**(k+1)
        for t in [pentagonal_number(k), pentagonal_number(-k)]:
            if (n-t) >= 0:
                partitions[n] = partitions[n] + coeff*partitions[n-t]
    return partitions

k=0
for i in compute_partitions(200):
    k=k+1
    print(k,i)

```

Current note has 0 direct children and 0 total descendants.

- [stretchy](#)
 - $\mathbb{Z}$
 - [Partitions of naturals](#)

And it has 6 siblings.

- [stretchy](#)
 - $\mathbb{Z}$
 - [Division in \$\mathbb{Z}\$](#)
 - [Euler totient function](#)
 - [gcd of integers](#)
 - [Primes in \$\mathbb{N}\$](#)
 - [Partitions of naturals](#)
 - [Prime-representing function](#)

