> ⓘ Info
>
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Exponential on $\text{Mat}_{\mathbb{C}}(2)$

exponential of JCF

> **Proposition:**
>
> $$e^{\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}} = \begin{cases} e^a \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} & \text{if } a = d \\ \begin{bmatrix} e^a & \frac{b(e^a - e^d)}{a-d} \\ 0 & e^d \end{bmatrix} & \text{if } a \neq d \end{cases}$$
>
> and thus we may also compute the flows on $GL_{\mathbb{C}}(2)$
>
> $$e^{t\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}} = \begin{cases} e^{ta} \begin{bmatrix} 1 & tb \\ 0 & 1 \end{bmatrix} & \text{if } a = d \\ \begin{bmatrix} e^{ta} & \frac{b(e^{ta} - e^{td})}{(a-d)} \\ 0 & e^{td} \end{bmatrix} & \text{if } a \neq d \end{cases}$$

$$\left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} \begin{bmatrix} e^{at} & bte^{ta} \\ 0 & e^{at} \end{bmatrix} = \left.\begin{bmatrix} ae^{at} & be^{ta} + abte^{ta} \\ 0 & ae^{at} \end{bmatrix}\right|_{t=0} = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$$

[1]

exponential given eigenvalues

> Let $A \in \text{Mat}_{\mathbb{C}}(2)$ have eigenvalues λ_1, λ_2. If $\lambda_1 = \lambda_2 = \lambda$ then
>
> $$e^A = e^{\lambda}((1-\lambda)I + A)$$
>
> and otherwise
>
> $$e^A = \frac{\lambda_1 e^{\lambda_2} - \lambda_2 e^{\lambda_1}}{\lambda_1 - \lambda_2} I + \frac{e^{\lambda_1} - e^{\lambda_2}}{\lambda_1 - \lambda_2} A$$

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1. Some explicit formulas for the matrix exponential - Automatic Control, IEEE Transactions on (umich.edu) ↩