Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
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$\mathfrak{sl}(2, \mathbb{C})$

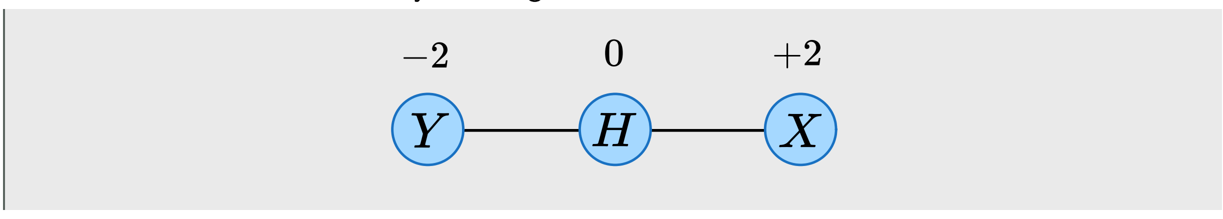
The Lie algebra $\mathfrak{sl}(2, \mathbb{C})$ is the set of all **2 x 2 complex matrices with trace $= 0$**. This space is spanned by

$$H := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{(traceless diagonal)}$$
$$X := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} (= E_{12})$$
$$Y := \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} (= E_{21})$$

with

$$[H, X] = 2X$$
$$[H, Y] = -2Y$$
$$[X, Y] = H$$

whose structure is understood by the diagram



Thus we have

$$\mathfrak{sl}_2\mathbb{C} \cong_{\mathsf{Alg}} \mathbb{C}\left\langle H, X, Y \middle| \begin{matrix} [H, X] = 2X \\ [H, Y] = -2Y \\ [X, Y] = H \end{matrix} \right\rangle$$

For each $n \in \mathbb{N}$ there exists an unique $n + 1$ dimensional $\mathsf{Vec}_{\mathbb{C}}$ irreducible representation

$$\mathfrak{sl}(2, \mathbb{C}) \to \mathsf{EndVec}_{\mathbb{C}}(Sym^n\mathbb{C})$$

which is the n-th symmetric power of the standard representation on $\mathbb{C}^2$.

$\mathsf{FinVec}_{\mathbb{C}}$ irreducible representations of $\mathfrak{sl}(2, \mathbb{C})$

Current note has 0 direct children and 0 total descendants.

- squishy
  - SL
    - $SL(2)(\mathbb{C})$
      - $\mathfrak{sl}(2, \mathbb{C})$

And it has 2 siblings.

- squishy
  - SL
    - $SL(2)(\mathbb{C})$
      - $\mathfrak{sl}(2, \mathbb{C})$
      - $PSL(2)(\mathbb{C})$