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Zariski closure of subgroups of $GL(n, \mathbb{C})$

Zariski closure of compact subgroups

Let $K \leq GL(n, \mathbb{C})$ be a compact subgroup and let $\overline{\overline{K}}$ be the Zariski closure of K in $GL(n, \mathbb{C})$ then K is a maximal compact subgroup of $\overline{\overline{K}}$.

[1]

- Let U be a compact subgroup of G containing K and suppose $K \neq U$.
- Then consider $u \in U \setminus K$ implying $Ku \cap K = \emptyset$.
 - Since Ku, K are both compact, there is a continuous $f : U \rightarrow \mathbb{R}$ such that

$$f|_K \equiv 1, f|_{Ku} \equiv 0$$

- By the Stone-Weirstrass theorem there is a polynomial $\phi \in \mathbb{C}[U]$ such that

$$\begin{aligned} \forall x \in U, |f(x) - \phi(x)| &< \frac{1}{4} \\ \implies \forall k \in K, |\phi(ku)| &< \frac{1}{4}, |\phi(k)| > \frac{3}{4} \end{aligned}$$

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Current note has 0 direct children and 0 total descendants.

- [stem](#)
 - $GL(n)(\mathbb{C})$
 - [Zariski closure of subgroups of \$GL\(n, \mathbb{C}\)\$](#)

And it has 1 siblings.

- [stem](#)
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1. <https://mathweb.ucsd.edu/~nwallach/lectures-2-207c-new-2.pdf> ↔