Info

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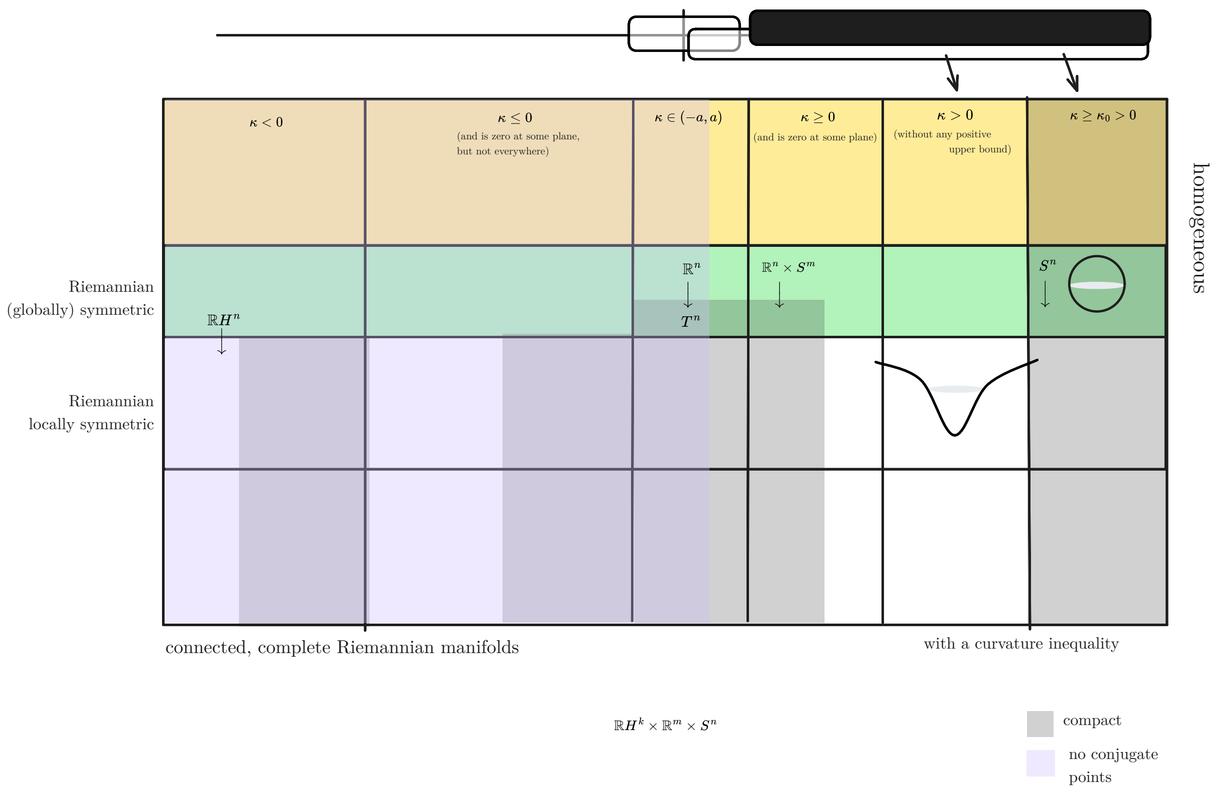
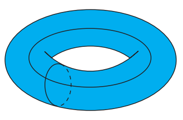
Complete Riemannian manifolds





completeness

(Ambrose) Let

$$\pi : \underbrace{(X, h)}_{\text{complete}} \to (M, g)$$

be a local isometry of **connected** Riemannian manifolds with (X, h) complete. Then (M, g) **is complete** and π is a Riemannian covering map.

Caution

If

$$\pi : (X, h) \to \underbrace{(M, g)}_{\text{complete}}$$

is a local isometry, then it **not** guaranteed that π is a covering.

Let

$$\pi : (X, h) \to (M, g)$$

be a Riemannian covering map between connected Riemannian manifolds. Then (M, g) is complete $\iff$ (X, h) is complete.

- One side is from

> **(Ambrose)** Let
>
> $$\pi : \underbrace{(X, h)}_{\text{complete}} \to (M, g)$$
>
> be a local isometry of **connected** Riemannian manifolds with (X, h) complete. Then (M, g) **is complete** and π is a Riemannian covering map.

- Conversely, if M is complete, lifts of geodesics on M are geodesics on X.

Let (M, g) be a connected Riemannian manifold and $p \in M$. If

$$\exp_p : T_pM \to M$$

is (well-defined and) a local diffeomorphism everywhere, then $\exp_p$ is a smooth covering map.

constant curvature

(Killing-Hopf) Let (M, g) be a complete, simply connected Riemannian $n \ge 2$-manifold with constant sectional curvature. Then M is isometric to one of

$$\mathbb{R}^n, (S^n, Rg), (\mathbb{R}H^n, Rg)$$

Corollary 12.5 (Characterization of Constant-Curvature Manifolds). *The complete, connected, n-dimensional Riemannian manifolds of constant sectional curvature are, up to isometry, exactly the Riemannian quotients of the form $\widetilde{M}/\Gamma$, where $\widetilde{M}$ is one of the constant-curvature model spaces $\mathbb{R}^n$, $\mathbb{S}^n(R)$, or $\mathbb{H}^n(R)$, and Γ is a discrete subgroup of $\operatorname{Iso}(\widetilde{M})$ that acts freely on $\widetilde{M}$.*

positive

(Bonnet-Myers) Let (M, g) be a complete connected Riemannian n-manifold and suppose there is a $R > 0$ such that

$$\mathrm{Ric}_g(v, v) \geq \frac{n-1}{R^2}$$

for all unit vectors $v \in T^{\mathrm{u}}(M, g)$. Then M is **compact** with

$$\mathrm{diam}(M, g) \leq \pi R$$

and $\pi_1(M)$ is finite.

Current note has 14 direct children and 19 total descendants.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold semi-Riemannian
      - Complete Riemannian manifolds
        - Integrating sectional curvature
        - Riemannian manifolds with negative curvature
          - Geodesic flow on Riemannian manifolds of negative curvature
          - Discrete subgroup of isometry group of a simply connected Riemannian manifold of negative curvature
          - Rigidity of marked length spectrum for Riemannian 2-manifolds of negative curvature
          - Equivalence of marked length spectrum, geodesic flow and boundary of universal cover of Riemannian manifolds with negative curvature
        - Riemannian manifolds with non-positive sectional curvature
          - Boundary of simply connected Hadamard manifolds
        - Cut locus in a complete Riemannian manifold
        - Exponential map for a complete Riemannian manifold
        - Geodesic completeness $\iff$ metric completeness of Riemannian manifolds
        - Geodesics in homotopy classes
        - Homogeneous Riemannian manifolds
        - Homogeneous and isotropic Riemannian manifolds
        - Poles of a complete Riemannian manifold
        - Riemannian symmetric spaces
        - Uniquely geodesic complete Riemannian manifolds
        - Riemannian homogeneous manifolds where $\exp_p$ is a diffeomorphism
        - Complete Riemannian manifolds without conjugate points

And it has 18 siblings.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold semi-Riemannian
      - Universal cover of semi-Riemannian manifolds
      - Curvature of a Riemannian manifold
      - Einstein manifolds
      - Exponential map from tangent bundle
      - Geodesics past conjugate points in a Riemannian manifold
      - Riemannian manifolds with ergodic geodesic flow
      - Isometries of a semi-Riemannian manifold
      - Laplacian on a Riemannian manifold
      - Parallel transport on a curve in a semi-Riemannian manifold
      - Conjugate points in a semi-Riemannian manifold
      - Quotients of Riemannian manifolds
      - Complete Riemannian manifolds
      - Riemannian connection
      - Riemannian submanifolds
      - Totally geodesic submanifolds of semi-Riemannian manifolds
      - Tangent bundle of a Riemannian manifold
      - Unit tangent bundle of a Riemannian manifold
      - Vector fields on Riemannian manifolds