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Discrete subgroups of

$$\begin{aligned} O^+(1, n) &\cong \operatorname{Isom}(\mathbb{R}\boldsymbol{H}^n) \\ &\cong \operatorname{Moeb}(B^n) \\ &\cong \operatorname{Moeb}(S^{n-1}) \end{aligned}$$

$\leftrightarrow$

hyperbolic n -manifolds/orbifolds

$\leftrightarrow$

convex n -dimensional polyhedron in $\mathbb{R}\boldsymbol{H}^n$ equipped with a cocomplete, subproper $\operatorname{Isom}(\mathbb{R}\boldsymbol{H}^n)$ -side-pairing

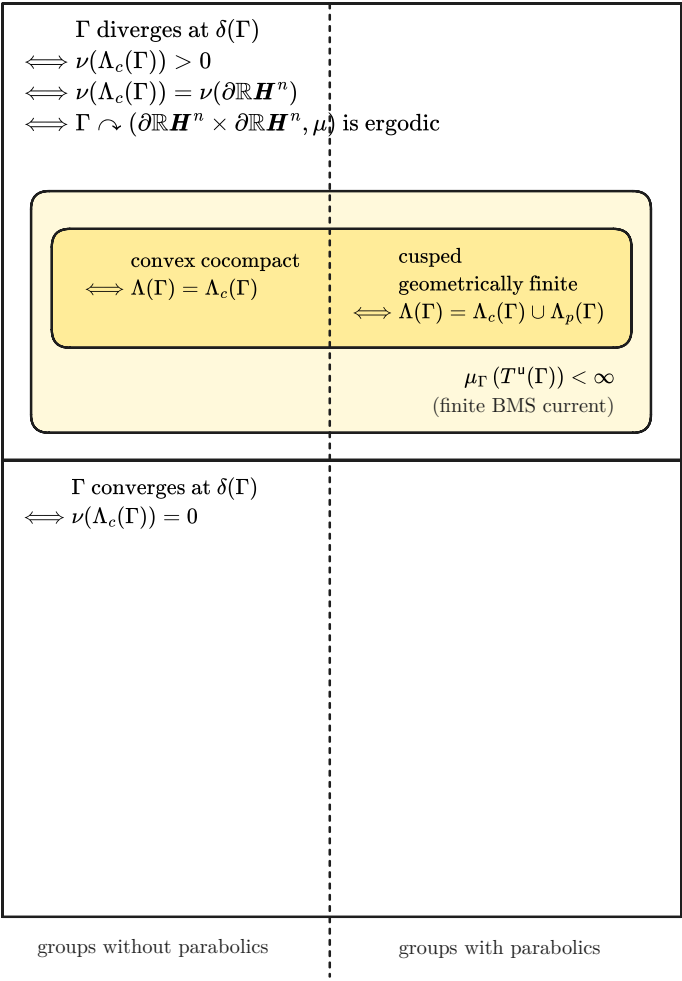
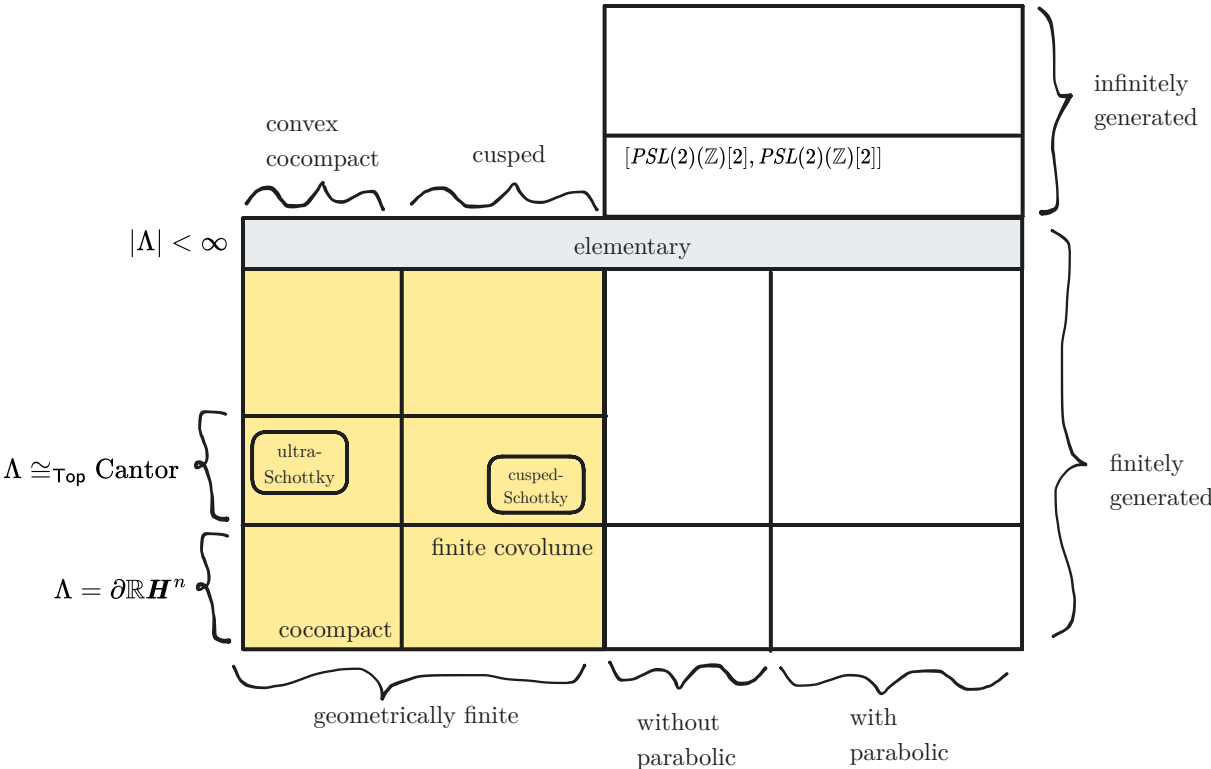
$\leftrightarrow$

uniformizable $(S^{n-1}, \operatorname{Moeb}(S^{n-1}))$ -orbifolds
($\mathbb{R}, \mathbb{C}$ -projective for $n = 2, 3$, conformally flat for $n > 3$)

Let

$$\Gamma \leq O^+(1, n)(\mathbb{R}) \cong \operatorname{Isom}(\mathbb{R}\boldsymbol{H}^n) \curvearrowright \mathbb{R}\boldsymbol{H}^n$$

be a discrete subgroup.



category and moduli space of hyperbolic manifolds

Definition. Hyperbolic manifolds

A $\mathbb{R}$ -hyperbolic manifold is a connected, [complete Riemannian manifold](#) with constant sectional curvature $\kappa \equiv 1$.

Let X be a $\mathbb{R}$ -hyperbolic n -manifold. Then there is a covering map

$$\pi_1(X) \hookrightarrow O^+(1, n)(\mathbb{R}) \curvearrowright \mathbb{R}\boldsymbol{H}^n$$

$$\downarrow$$
$$X$$

where the Deck transformations act by isometries $\pi_1(X) \hookrightarrow O^+(1, n)(\mathbb{R})$.

Let

$$\mathbb{R}\boldsymbol{H}^n\text{-Riem}$$

be the category of hyperbolic manifolds of dimension n and let

$$\{\text{torsion free discrete subgroups of } O^+(1,n)(\mathbb{R})\}$$

be the category of torsion free discrete subgroups of $O^+(1,n)(\mathbb{R})$ with morphisms

$$\mathfrak{c}_g:\Gamma_1\rightarrow\Gamma_2$$

being conjugacy homomorphisms for $g\in O^+(1,n)(\mathbb{R})$.

Then

$$\begin{aligned} \{\text{torsion free discrete subgroups of } O^+(1,n)(\mathbb{R})\} &\rightarrow \mathbb{R}\boldsymbol{H}^n\text{-Riem} \\ \Gamma &\mapsto \Gamma\backslash\mathbb{R}\boldsymbol{H}^n \end{aligned}$$

is a functor, that produces an equivalence of categories?

We have the forgetful functor

$$\mathbb{R}\boldsymbol{H}^n\text{-Riem}\rightarrow {}^n\text{Man}$$

We have subcategories

$$\mathbb{R}\boldsymbol{H}^n\text{-Riem}\supset\mathbb{R}\boldsymbol{H}^n\text{-FinRiem}\supset\mathbb{R}\boldsymbol{H}^n\text{-cptRiem}$$

[1]

and a map

$$\text{vol}:\mathbb{R}\boldsymbol{H}^n\text{-FinRiem}\rightarrow [0,\infty)$$

upto isomorphism

We have

$$\{\text{torsion free discrete subgroups of } O^+(1,n)(\mathbb{R})\}_{\cong}\leftrightarrow \mathbb{R}\boldsymbol{H}^n\text{-Riem}_{\cong}\rightarrow {}^n\text{Man}_{\cong}$$

[2]

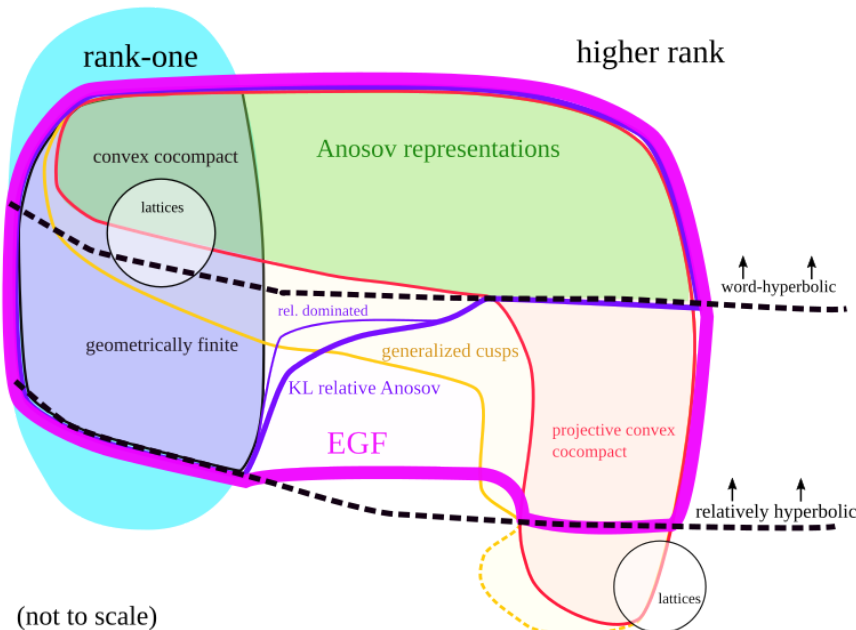
Current note has 11 direct children and 17 total descendants.

- [stem](#)
 - [O+ R1 n](#)
 - [Discrete subgroups of \$O^+\(1,n\)\$ ↔ hyperbolic \$n\$ -manifolds/orbifolds](#)
 - [Convex cocompact discrete subgroups](#)
 - [Combination of discrete subgroups of \$O^+\(1,n\)\(\mathbb{R}\)\$](#)
 - [Exponential sum of a discrete subgroup of \$\text{Isom}\(\mathbb{R}\boldsymbol{H}^n\)\$](#)
 - [RepDF\(\$F_g,O^+\(1,n\)\(\mathbb{R}\)\$ \)](#)
 - [Measure homology on a hyperbolic manifold](#)
 - [Discrete subgroups with limit set a discontinuum](#)
 - [Limit sets of discrete subgroups](#)
 - [Conical limit set](#)
 - [Discrete subgroups with limit set a \$k-1\$ -sphere](#)
 - [Counting orbits of discrete subgroups of \$O^+\(1,n\)\(\mathbb{R}\)\$ in \$\mathbb{R}\boldsymbol{H}^n\$](#)
 - [Rigidity of \$\mathbb{R}\$ -hyperbolic manifolds](#)
 - [Using ergodic current on \$\partial^2\mathbb{R}\boldsymbol{H}^n\$](#)
 - [Rigidity using measure homology](#)
 - [Rigidity of lattices using quasi-conformal mappings](#)
 - [Schottky](#)
 - [Classical Schottky subgroups of \$O^+\(1,n\)\$](#)
 - [Schottky subgroups of \$O^+\(1,n\)\$](#)
 - [RepDF\(\$\pi_1\(T_g^2,\bullet\),O^+\(1,n\)\(\mathbb{R}\)\$ \)](#)

And it has 2 siblings.

- [stem](#)
 - [O+ R1 n](#)
 - [Subgroups of \$O^+\(1,n\)\(\mathbb{R}\)\$](#)
 - [Discrete subgroups of \$O^+\(1,n\)\$ ↔ hyperbolic \$n\$ -manifolds/orbifolds](#)

1. [Hyperbolic manifolds according to Thurston and Jørgensen - 129-1.pdf](#)↔



2.