ⓘ Info

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$\boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n})$

We have

$$\boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n})$$

which is a 2^n dimensional associative $\mathbb{R}$-algebra, $\mathbb{R}$-generated by $e_i \dots e_k$ such that

$$\begin{aligned} e_i e_i &= -1 \\ e_i e_j &= -e_j e_i \end{aligned}$$

with the inclusions

$$\begin{aligned} \mathbb{R}^n &\hookrightarrow \boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n}) \\ e_i &\mapsto e_i \end{aligned}$$

which we see as a consequence of extending $\mathbb{R}^n$ into the algebra and

$$\begin{aligned} \mathbb{R} &\hookrightarrow \boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n}) \\ 1 &\mapsto 1 \end{aligned}$$

We interpret

$$\begin{aligned} \mathbb{R}\oplus\mathbb{R}^n &\hookrightarrow \boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n}) \\ (1,0) &\mapsto 1 \\ (0,e_i) &\mapsto e_i \end{aligned}$$

as the *CR operator embedding* as we now may see

$$\partial_0 + \sum_{i=1}^{n} e_i \partial_i$$

as acting on functions $U \subseteq \mathbb{R}^{n+1} \to \boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n})$

$\boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n})$	**as an algebra**	$\mathbb{R}\oplus\mathbb{R}^n \hookrightarrow \boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ $ $(1,0)\mapsto 1$ $(0,e_i)\mapsto e_i$	**the CR operator**
$n=1$	$\mathbb{R}\langle 1, e_1\rangle \cong \mathbb{C}$ complex numbers	is a $\mathbb{R}$-vector space **isomorphism** because $1+1=2^1$	$\partial_0 + e_1\partial_1 = \dfrac{\partial}{\partial \bar{z}}$
$n=2$	$\mathbb{R}\langle 1, e_1, e_2, e_1e_2\rangle \cong \mathbb{H}$		$\partial_0 + e_1\partial_1 + e_2\partial e_2$

[1]

[2]

- A function $f : U \subseteq \mathbb{R}^{n+1} \to \boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n})$ is *left-monogenic on U*

$$\left(\partial_0 + \sum_{i=1}^{n} e_i \partial_i\right) f = 0 \text{ on } U$$

  - (Integral theorem)

$$\int_{\partial C} \mathrm{d}\sigma f = 0$$

  - (Integral formula)

$$x \in \mathrm{int}(C) \implies \frac{1}{a_{n+1}} \int_{\partial C} \frac{1}{y-x} \mathrm{d}\sigma_y f(y) = f(x)$$

[3]

Current note has 0 direct children and 0 total descendants.

- squishy
  - $\boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n})$

And it has 30 siblings.

- squishy
  - $SL(2)(\mathbb{R}) \curvearrowright \mathbb{R}^2$
  - $SL(2)(\mathbb{R}) \curvearrowright H^2_{\mathrm{U}}$
  - $\mathrm{Mat}_{\mathbb{C}}(2)$
  - $A_{r}{}_{\text{'-}\langle z\mapsto \lambda z\rangle}\backslash^{\mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}}$
  - $\mathbb{C}\setminus\{0\}_{\text{'-}\langle z\mapsto z+1\rangle}\backslash^{\mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}}$
  - $\mathbb{C}\setminus\{0,1\}_{\text{'-}PSL(2)(\mathbb{Z})[2]}\backslash^{\mathbb{R}\boldsymbol{H}^2_{\mathrm{U}}}$
  - $\mathbb{R}\ltimes_\theta \mathbb{C}^2$
  - $\boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n})$

- $\mathbb{C}^n$
- $U(\mathbb{C} \setminus \{0\})$
- $PSL(2)(\mathbb{C}) \curvearrowright \mathbb{C}P^1 \cong_{\mathsf{Man}} S^2$
- $\mathbb{C}P^n$
- Farey graph, complex and tree
- GL
- Heisenberg group
- Incomplete hyperbolic polygon mod side-pairing
- $\mathbb{R}\boldsymbol{H}^2$, $\mathbb{R}H^2_{\mathrm{U}}$, D^2
- $\mathbb{R}\boldsymbol{H}^n$
- D_n lattice
- Matrices
- $\mathsf{Mat}_{\mathbb{R}}(2)$
- Mobius endomorphisms
- Elementary group with finite and infinite sided Dirichlet polyhedra
- Polynomials
- $T^*S^1 \ncong_{\mathsf{Sym}} (\mathbb{R}^2 \setminus \{0\}, \mathrm{d}x \wedge \mathrm{d}y)$
- $Z\left(\sum_{1 \leq j \leq n} z_j^2 - 1\right)(\mathbb{C}) \cong_{\mathsf{Sym}} T^*S^{n-1}$
- SL
- SO
- Seifert–Weber dodecahedral 3-manifold
- $U(n, \mathbb{C})$

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1. 2403-Clifford-Analysis.pdf ↩
2. 0303339.pdf (arxiv.org) ↩
3. 2015_hds_fs_Introductory_Clifford_Analysis.pdf ↩