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# Distributional derivatives

## distributional derivatives of $L^1_{\text{loc}}$ functions

Definition. Distributional derivatives of  $L^1_{\text{loc}}$  functions

Let  $U(\text{open}) \subseteq \mathbb{R}^n$  be an open set and  $f \in L^1_{\text{loc}}(U)$ . Then  $w \in L^1_{\text{loc}}$  is a **distributional derivative** of  $f$  in  $\hat{x}_j$  direction

$$\int \partial_j f = w \text{ if } \forall \varphi \in \mathcal{C}_c^\infty(U), \int_U w \varphi = - \int_U f \partial_j \varphi$$

Current note has 4 direct children and 4 total descendants.

- stem
  - subobjects of and functions on  $\mathbb{R}^n, S^n, \mathbb{C}^n$  and its quotients  $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$ 
    - Differentiable functions
      - Distributional derivatives
        - Fractional Sobolev spaces
        - $H^1(U)$
        - $H^1_{\text{loc}}$
        - $H^1_{\text{loc}}(U)$  for  $U \subseteq \mathbb{R}^2$

And it has 10 siblings.

- stem
  - subobjects of and functions on  $\mathbb{R}^n, S^n, \mathbb{C}^n$  and its quotients  $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$ 
    - Differentiable functions
      - Functions  $(a, b) \rightarrow \mathbb{R}$  differentiable at a point
      - Comparing a function and it derivative
      - Absolutely continuous functions on  $[a, b] \leftrightarrow \int_{[a, -]} (L^1[a, b])$
      - Distributional derivatives
      - Double derivative/Laplace operator on the circle
      - Fractional derivative
      - Infinite limit of derivatives
      - Space of continuous and continuously differentiable functions on  $\mathbb{R}$
      - Derivative of maps  $\mathbb{R}^n \rightarrow \mathbb{R}^m$
      - Zooming of a map  $\mathbb{R}^n \rightarrow \mathbb{R}^m$