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Branch points and ramification index of holomorphic functions

Definition. Branch points and ramification index of holomorphic functions

Let $\Omega \subseteq \mathbb{C}$ be a connected open set and

$$f : \Omega \rightarrow \mathbb{C}$$

be a non-constant holomorphic function. Then zeros of f' , that is, $(f')^{-1}(0)$ are the set of *critical points* of f and in particular this set has no limit points in Ω as f is holomorphic.

The **ramification index** of f is equivalently

- Around a small disk $B(z_0, r)$ of such a zero point $z_0 \in (f')^{-1}(0)$ which does not contain any other zeros, we have the winding number of the curve $f(\gamma)$
- order of the zero z_0 of f'

If the ramification index at z_0 is greater than 1 then z_0 is called a **ramification point** of f , the *critical value* $f(z_0)$ is called an **(algebraic) branch point** of f .

Equivalently, z_0 is a ramification point of f if

$$f(z) = f(z_0) + (z - z_0)^k g(z), \; k > 1$$

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