Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
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Arnold cat map $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \curvearrowright T^2$

Arnold's cat map - Wikipedia

powers

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{2} = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{3} = \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{4} = \begin{bmatrix} 34 & 21 \\ 21 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{5} = \begin{bmatrix} 89 & 55 \\ 55 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{6} = \begin{bmatrix} 233 & 144 \\ 144 & 89 \end{bmatrix}$$

The elements of the powers are elements of the Fibonacci sequence

Definition. Fibonacci sequence

The Fibonacci sequence is defined by the recurrence relation

$$\begin{aligned} F(0) &= 0 \\ F(1) &= 1 \\ F(n+2) &= F(n) + F(n+1) \end{aligned}$$

which looks like

$$0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{20} = \begin{bmatrix} 165580141 & 102334155 \\ 102334155 & 63245986 \end{bmatrix}$$

A square root in $GL(2)(\mathbb{Z})$ is

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{2} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

and because

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n} = \begin{bmatrix} F(n+1) & F(n) \\ F(n) & F(n-1) \end{bmatrix}$$

for all $n \geq 0$ where $F(n)$ is the Fibonacci sequence

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we conclude

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{n} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{2n} = \begin{bmatrix} F(2n+1) & F(2n) \\ F(2n) & F(2n-1) \end{bmatrix}$$

dynamics

“Anatole Katok, Boris Hasselbaltt - Introduction to the Modern Theory of Dynamical Systems-Cambridge University Press (1996).pdf#page=64&rect=22,148,412,570” could not be found.

“Anatole Katok, Boris Hasselbaltt - Introduction to the Modern Theory of Dynamical Systems-Cambridge University Press (1996).pdf#page=64&rect=21,33,414,146” could not be found.

https://www.desmos.com/calculator/dbkv88udsb

mapping torus

#wiki/Man/R/3

The mapping torus has Sol-geometry.

Current note has 0 direct children and 0 total descendants.

- squishy
  - Matrices
    - $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$
- stretchy
  - $GL(2)(\mathbb{Z}) \curvearrowright T^2$
    - Arnold cat map $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \curvearrowright T^2$

And it has 3 siblings.

- stretchy
  - $GL(2)(\mathbb{Z}) \curvearrowright T^2$
    - Arnold cat map $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \curvearrowright T^2$
    - $SL(2)(\mathbb{Z}) \curvearrowright T^2$
    - Suspension flows of $SL(2)(\mathbb{Z}) \curvearrowright T^2$