

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on September 6, 2026 7:06:13 PM, was last modified on September 6, 2026 11:57:40 PM, and was exported on September 7, 2026 3:24:54 PM.

Limit of a sequence of linear maps

sequence of bounded linear maps between Banach spaces

Let X, Y be Banach spaces and $\{T_n\}_{n \geq 1} \subseteq \mathcal{B}(X, Y)$ be a sequence of bounded linear maps such that for each $x \in X$, the sequence $T_n(x)$ converges in Y . Then

$$x \mapsto \lim_{n \rightarrow \infty} T_n(x)$$

is a bounded linear map.

Let

$$S_n : X \rightarrow Y, S_\infty : X \rightarrow Y$$

be bounded linear operators between Banach spaces with $\|S_n\|_{X \rightarrow Y} \leq M$ and

$$S_n f \xrightarrow{n \rightarrow \infty, \| \cdot \|_Y} S_\infty f$$

for all $f \in D(\text{dense}) \subset X$. Then

$$g \in X \implies S_n g \xrightarrow{n \rightarrow \infty, \| \cdot \|_Y} S_\infty g$$

Consider $g \in X$. For any $\epsilon > 0$, consider $f_\epsilon \in D$ such that

$$\|g - f_\epsilon\|_X < \epsilon$$

- Choose N_ϵ such that

$$n > N_\epsilon \implies \|S_n f_\epsilon - S_\infty f_\epsilon\| < \epsilon$$

.

$$\begin{aligned} \|S_n g - S_\infty g\| &\leq \|S_n(g - f_\epsilon)\| + \|S_n f_\epsilon - S_\infty f_\epsilon\| + \|S_\infty(f_\epsilon - g)\| \\ &\leq \underbrace{\|S_n\|}_{\leq M} \underbrace{\|(g - f_\epsilon)\|_Y}_{\leq \epsilon} + \underbrace{\|S_n f_\epsilon - S_\infty f_\epsilon\|}_{\leq \epsilon} + \|S_\infty\| \underbrace{\|(f_\epsilon - g)\|}_{\leq \epsilon} \\ &\leq \epsilon(M + \|S_\infty\| + 1) \end{aligned}$$

Current note has 0 direct children and 0 total descendants.

- [structures based on sets](#)
 - [Topological spaces](#)
 - [Topological vector spaces over \$\mathbb{R}\$ or \$\mathbb{C}\$](#)
 - [map](#)
 - [Limit of a sequence of linear maps](#)

And it has 3 siblings.

- [structures based on sets](#)
 - [Topological spaces](#)
 - [Topological vector spaces over \$\mathbb{R}\$ or \$\mathbb{C}\$](#)
 - [map](#)
 - [Image of balls under linear map between topological vector spaces](#)
 - [Limit of a sequence of linear maps](#)
 - [Uniform boundedness of a famly of linear maps](#)