Info

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Riemann surfaces

Definition. Connected complex $\mathbb{C}$-manifolds of $\mathbb{C}$-dimension 1 are called **Riemann surfaces**.

We have

- Classification of complex 1-manifold
- sett.Man.C 1.T C complex tangent space
- Holomorphic functions on a complex 1-manifold holomorphic functions
  - Non-constant holomorphic maps between Riemann surfaces locally looks like w^k and are open maps
  - Global holomorphic functions on a complex 1-manifold
- Differential forms on complex 1-manifolds differential forms
  - Stroke's theorem
  - de Rham
  - ...
- Meromorphic functions on a Riemann surface meromorphic functions
- Vector bundles on complex 1-manifolds
  - line bundles

Current note has 16 direct children and 21 total descendants.

- stem
  - Objects stemming from $\mathbb{R}$-spaces of dimension 2
    - Riemann surfaces
      - Vector bundles on complex 1-manifolds
      - Classification of complex 1-manifold
      - Riemann's existence of holomorphic maps between compact Riemann surfaces
      - Divisors on compact Riemann surfaces
      - Fluid
      - Meromorphic differentials on Riemann surfaces
      - Holomorphic functions on a complex 1-manifold
        - Global holomorphic functions on a complex 1-manifold
        - Global algebraic functions on a complex 1-manifold
        - Non-constant holomorphic maps between Riemann surfaces locally looks like w^k and are open maps
      - Holomorphic maps of type $0 \overset{3}{\to} 0$
      - Rational maps
      - Hurwitz scheme
      - Moduli space of $\mathbb{C}$-manifolds
      - Meromorphic functions on a Riemann surface
      - Normalization of plane projective $\mathbb{C}$-curves and Riemann surfaces
      - Riemann surfaces defined over a number field
      - Hyperelliptic Riemann surfaces
      - T C
        - $\boldsymbol{Sym}^2(T^{(1,0)}X) \twoheadrightarrow X$
        - Differential forms on complex 1-manifolds
      - Riemann surfaces with a holomorphic 1-form
    - Objects stemming from $PSL(2)(\mathbb{R})$
      - Fuchsian groups $\Gamma <_{\mathsf{d}} PSL(2)(\mathbb{R})$ $\leftrightarrow$ oriented hyperbolic 2-manifold/orbifolds $\to$ Riemann surfaces with branching signature
        - Cocompact torsion-free Fuchsian groups

And it has 4 siblings.

- stem
  - Objects stemming from $\mathbb{R}$-spaces of dimension 2
    - Riemann surfaces
    - $\mathbb{C}$-analytic spaces of dimension 1
    - Riemann quasiconformal surfaces
    - Teichmuller space of surfaces