Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on December 28, 2025 10:31:37 PM,
was last modified on May 24, 2026 6:44:41 PM, and
was exported on September 7, 2026 2:50:32 PM.

$$\begin{bmatrix} 2 & \\ & \frac{1}{2} \end{bmatrix}$$

$(x, y) \mapsto (2x, 2^{-1}y)$

# on $\mathbb{R}^2 \setminus \{(0,0)\}$

On $\mathbb{R}^2 \setminus \{(0,0)\}$ we consider the linear map

$$(x, y) \mapsto (2x, 2^{-1}y)$$

given by

$$A := \begin{bmatrix} 2 & \\ & \frac{1}{2} \end{bmatrix}$$

We have the following forward orbit

$$(x, y) \mapsto (2x, 2^{-1}y) \mapsto (2^2x, 2^{-2}y) \mapsto \dots (2^n x, 2^{-n}y) \mapsto \dots$$

https://www.desmos.com/calculator/8ipwueqauo

- every orbit is discrete

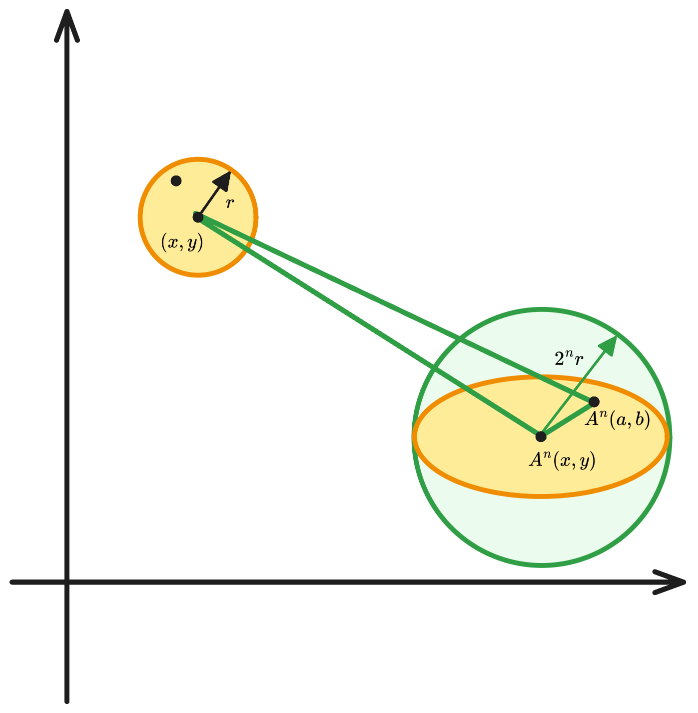
> **Proposition:** For any $(x, y) \in \mathbb{R}^2 \setminus \{(0,0)\}$ there is a $r$ small enough such that
> $$\{A^n(B_r(x, y)) | n \in \mathbb{Z}\}$$
> is disjoint.

https://www.desmos.com/calculator/k1z1pnbl0e

Let

$$r(x, y) := \frac{\|(x, y)\|}{5}$$

- We know
$$\|A^n(x, y)\| \leq 2^n \|(x, y)\|$$
- Let
$$(a, b) \in B_r(x, y)$$
- For $n \in \mathbb{Z} \setminus \{1\}$ we have




- $$\begin{aligned} \|(x, y) - A^n(a, b)\| &\geq \|(x, y) - A^n(x, y)\| - \underbrace{\|A^n(a, b) - A^n(x, y)\|}_{\leq 2^n r} \\ &\geq \left\| \left( (2^n - 1)x, \left(1 - \frac{1}{2^n}\right) y \right) \right\| - 2^n r \\ &\geq (2^{|n|} - 1) \underbrace{\|(x, y)\|}_{=5r} - 2^n r \\ &= r \underbrace{(5(2^{|n|}) - 2^n - 5)}_{\geq 2} \geq 2r \end{aligned}$$

orbits of $(0, x)$ and $(y, 0)$ does not satisfy

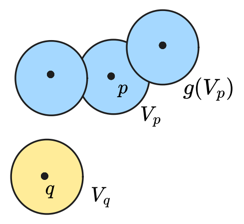
**Definition.** coHausdorff actions

Let $G$ be a group and

$$G \to \mathrm{Homeo}(Y)$$

be an *action by homeomorphisms* on a topological space $Y$. The action is called **coHausdorff** if it satisfies *either* of

- $\Gamma \backslash X$ is Hausdorff
- if $p, q \in X$ are not in same $\Gamma$-orbit then there exists neighborhoods $V_p$ of $p$ and $V_q$ of $q$ such that
$$\forall g \in \Gamma, g(V_p) \cap V_q = \emptyset$$




Current note has 0 direct children and 0 total descendants.

- squishy
  - Matrices
    - $\begin{bmatrix} 2 & \\ & \frac{1}{2} \end{bmatrix}$

And it has 4 siblings.

- squishy
  - Matrices
    - $\begin{bmatrix} 2 & \\ & \frac{1}{2} \end{bmatrix}$
    - $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$
    - $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
    - $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$