ⓘ Info

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Kleinian group with $\Lambda(\Gamma) \neq \partial\mathbb{R}\boldsymbol{H}^3$

$\leftrightarrow$

disjoint union of Riemann surfaces with branching signature

Definition. Ω-Kleinian groups

Let $\Gamma <_{\mathrm{d}} PSL(2)(\mathbb{C})$ be a Kleinian group with $\Lambda(\Gamma) \neq \partial\mathbb{R}\boldsymbol{H}^3$. Then

- the non-empty open set

$$\Omega(\Gamma) := \partial\mathbb{R}\boldsymbol{H}^3 \setminus \Lambda(\Gamma)$$

  is the **domain of discontinuity**
- the set of points in $\Omega(\Gamma)$ at which the action $\Gamma \curvearrowright \Omega(\Gamma)$ is freely discontinuous is called the **free regular set** ${}^{\circ}\Omega(\Gamma)$
- a **component** of Γ is a connected component of $\Omega(\Gamma)$
- a component $\Omega_0 \subseteq \Omega(\Gamma)$ is **invariant** if $\Gamma\Omega_0 = \Omega_0$

E.3. In general, we say that a set *Y* is *precisely invariant* under the subgroup *H* in *G*, if
(i) $H = \mathrm{Stab}_G(Y)$, and
(ii) $g(Y) \cap Y = \varnothing$ for all $g \in G - H$.
Where there is no danger of confusion, we will simply say that *Y* is precisely invariant under *H*, or, even more simply, that *Y* is precisely invariant.
Note that $x \in {}^{\circ}\Omega$ if and only if there is a neighborhood *U* of *x* which is precisely invariant under the identity in *G*.

Let Γ be a Ω-Kleinian group. Then $\Omega(\Gamma) \setminus {}^{\circ}\Omega(\Gamma)$ is discrete and ${}_{\Gamma}\backslash^{\Omega(\Gamma)}$ is a complex 1-manifold.

Let Γ be a Ω-Kleinian group.

- For a connected component $Y \subseteq {}_{\Gamma}\backslash^{\Omega(\Gamma)}$, consider a connected component $T \subseteq \pi^{-1}(Y)$.
  - Then T is *precisely* $\mathrm{stab}_{\Gamma}(T)$*-invariant*.
- Conversely, let $T \subseteq \Omega(\Gamma)$ be any connected component which is *precisely* $\mathrm{stab}_{\Gamma}(T)$*-invariant*.
  - Then T is a connected component of $\pi^{-1}(\pi(T))$.

An open, connected, non-empty subset $T \subseteq \Omega(T)$ which is *precisely* $\mathrm{stab}_{\Gamma}(T)$*-invariant* is a Γ-**panel**.

Definition. Definition

Let $\Gamma <_{\mathrm{d}} PSL(2)(\mathbb{C})$ be a Kleinian group with $\Lambda(\Gamma) \neq \partial\mathbb{R}\boldsymbol{H}^3$ and an *invariant component* Δ. Then (Γ, Δ) is a **function group** if ${}_{\Gamma}\backslash^{\Delta}$ is a marked Riemann surface of finite type. The quotient map

$$\begin{array}{l} \Delta \\ \downarrow {\scriptstyle \Gamma\backslash} \\ {}_{\Gamma}\backslash^{\Delta} \quad \cong_{\mathsf{Man}_{\mathbb{C}}} X \setminus \{x_1, \ldots, x_m\} \hookrightarrow \underbrace{X}_{\text{compact}} \end{array}$$

is a ramified covering. The **signature** of (Γ, Δ) is the genus of X, the number of punctures n and the ramification orders $(g, m; v_1, \ldots, v_n)$.

[1]

Current note has 0 direct children and 0 total descendants.

- stem
  - Objects stemming from $PSL(2)(\mathbb{C})$
    - Kleinian groups $\Gamma <_{\mathrm{d}} PSL(2)(\mathbb{C})$ $\leftrightarrow$ Hyperbolic 3-manifolds/orbifolds ${}_{\Gamma}\backslash^{\mathbb{R}\boldsymbol{H}^3}$
      - Kleinian group with $\Lambda(\Gamma) \neq \partial\mathbb{R}\boldsymbol{H}^3$

And it has 7 siblings.

- stem
  - Objects stemming from $PSL(2)(\mathbb{C})$
    - Kleinian groups $\Gamma <_{\mathrm{d}} PSL(2)(\mathbb{C})$ $\leftrightarrow$ Hyperbolic 3-manifolds/orbifolds ${}_{\Gamma}\backslash^{\mathbb{R}\boldsymbol{H}^3}$
      - Finitely generated, free Kleinian groups
      - 2-generated Kleinian groups
      - Finitely generated Kleinian groups $\leftrightarrow$ topologically tame oriented hyperbolic 3-manifolds/finite degree orbifold covers of
      - Building hyperbolic 3-manifolds by gluing polyhedra
      - Kleinian group with $\Lambda(\Gamma) \neq \partial\mathbb{R}\boldsymbol{H}^3$
      - Thick-think decomposition of hyperbolic 3-manifolds

- Kleinian ultra-Schottky groups $\leftrightarrow$ convex cocompact hyperbolic interior of 3-handlebodies

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1. Bernard Maskit - Kleinian Groups (Grundlehren Der Mathematischen Wissenschaften), p.84 ↩