Info

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Chain complex of differential forms on a smooth manifold

Definition. Chain complex of differential forms on a smooth manifold

For any open set $U \subseteq M$ of a smooth manifold M the chain complex of $\mathbb{R}$-vector spaces (Ω^*, d^*)

$$0 \to \Omega^0(U) \xrightarrow{\mathrm{d}^0} \cdots \xrightarrow{\mathrm{d}^{k-1}} \Omega^k(U) \xrightarrow{\mathrm{d}^k} \cdots \xrightarrow{\mathrm{d}^{n-1}} \Omega^n(U) \to 0$$

where the maps d^k are restricted to $\Omega^k(U)$ is called the **de Rham complex for** U and its k-th cohomology $\mathbb{R}$-vector space

$$H^k(\Omega^\bullet(U)) := \frac{\ker(\mathrm{d}^k)}{\mathrm{im}\,(\mathrm{d}^{k-1})}$$

the **de Rham cohomology for** U.

For $k < 0$ and $k > \dim M$, $H^k_{dR} = 0$.

If M is a connected smooth manifold then

$$H^0_{dR}(M) \cong \mathbb{R}$$

(Homotopy invariance of $H^k(\Omega^\bullet)$) Given two continuously homotopic, smooth maps

$$f, g : M \to N$$

between manifolds possibly with boundary, we can always produce a smooth homotopy between them. Two such smoothly homotopic maps induce equal homomorphisms in the cohomology of differential forms

$$f^* = g^* : \Omega^\bullet(N) \to \Omega^\bullet(M)$$

In particular, if

$$\alpha : M \to N, \beta : N \to M$$

is a smooth homotopy equivalence, then α^*, β^* induce isomorphisms in the cohomology of differential forms.

isomorphism with singular cohomology

Current note has 1 direct children and 1 total descendants.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Differential forms on smooth manifolds
          - Chain complex of differential forms on a smooth manifold
            - Homotopy invariance of cohomology of differential forms

And it has 7 siblings.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Differential forms on smooth manifolds
          - Differential 1-forms on smooth manifolds
          - Chain complex of differential forms on a smooth manifold
          - Exterior derivative operator on space of smooth differential forms
          - Functoriality of Forms^k on smooth manifolds
          - The integral map on space of differential forms
          - Three different relative chain complexes of differential forms
          - The sheaf of differential forms on open subsets of manifolds