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Left-invariant Lagrangian on a Lie group

- Lagrangian

Theorem 2.2 (Poincaré [18]). *Let $g\colon [0,1] \rightarrow G$ be a solution to the Euler–Lagrange equations for L , thus extremizing the action functional*

$$A(g) = \int_0^1 L(g(t), \dot{g}(t))dt.$$

Then the path $\xi\colon [0,1] \rightarrow \mathfrak{g}$ defined by (7) fulfills the Euler–Poincaré equation

(8)
$$\frac{d}{dt}D\ell(\xi) - \operatorname{ad}^*_\xi D\ell(\xi) = 0,$$

*where ad^*_ξ is the co-adjoint infinitesimal action of ξ on $\mathfrak{g}^*$, defined by*

(9)
$$\langle \operatorname{ad}^*_\xi \mu, \zeta \rangle = \langle \mu, [\xi, \zeta] \rangle, \quad \forall \zeta \in \mathfrak{g}.$$

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