Info

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Spin manifolds

Definition. Spin structure on a Riemannian vector bundle

Let

$$E \to M$$

be a oriented Riemannian $\mathbb{R}^n$-vector bundle over M and let $P_{SO}(E)$ be its bundle of oriented orthonormal frames.

- For $n \geq 3$, we have the universal covering homomorphism

  $$\mathrm{Spin}_n : \mathit{Spin}(n) \to SO(n)$$

  with kernel $\{I, -I\} \cong \mathbb{Z}_2$. A **spin structure** on E is a principal $\mathit{Spin}(n)$-bundle $P_{\mathit{Spin}}(E)$ together with a 2-sheeted covering

  $$\zeta : P_{\mathit{Spin}}(E) \to P_{SO}(E)$$

  such that $\zeta(pg) = \zeta(p)\mathrm{Spin}_n(g)$ for all $p \in P_{\mathit{Spin}}(E)$ and $g \in \mathit{Spin}(n)$
- For $n = 2$

[1]

Definition. A **spin manifold** is an oriented Riemannian manifold M with a spin structure on its tangent bundle $TM \to M$.

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - Spin $\mathbb{R}$-manifold

And it has 20 siblings.

- structures based on sets
  - manifolds
    - Closed aspherical manifolds
    - $\mathbb{C}$-manifold
    - Hermitian manifolds
    - Kahler manifolds
    - Kahler manifolds of complex dimension 2, AKA Kahler surfaces
    - smooth $\mathbb{R}$-manifold
    - Principal bundles over smooth manifolds
    - Manifold with a flow
    - $\mathbb{R}$-manifold group (Lie groups)
    - Homogeneous manifold
    - Finsler manifold
    - $\mathbb{R}$-manifold semi-Riemannian
    - Sub-Riemannian manifolds
    - $\mathbb{R}$-manifold Poisson
    - $\mathbb{R}$-manifold symplectic
    - Multisymplectic manifolds
    - Spin $\mathbb{R}$-manifold
    - $\mathbb{R}$-manifold topological
    - Toric manifolds
    - Manifolds with compatible triples

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1. H. Blaine Lawson, Marie-Louise Michelsohn - Spin geometry -Princeton University Press (1989), page 90 ↩